

# **control 2.1.55**

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Control Systems Package for GNU Octave

**Lukas F. Reichlin**

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# 1 Function Reference

## 1.1 Examples and Demos

### 1.1.1 BMWengine

```
sys = BMWengine () [Function File]
sys = BMWengine ("scaled") [Function File]
sys = BMWengine ("unscaled") [Function File]
```

Model of the BMW 4-cylinder engine at ETH Zurich's control laboratory.

#### OPERATING POINT

Drosselklappenstellung	alpha_DK = 10.3 Grad
Saugrohrdruck	p_s = 0.48 bar
Motordrehzahl	n = 860 U/min
Lambda-Messwert	lambda = 1.000
Relativer Wandfilminhalt	nu = 1

#### INPUTS

U_1 Sollsignal Drosselklappenstellung	[Grad]
U_2 Relative Einspritzmenge	[-]
U_3 Zuendzeitpunkt	[Grad KW]
M_L Lastdrehmoment	[Nm]

#### STATES

X_1 Drosselklappenstellung	[Grad]
X_2 Saugrohrdruck	[bar]
X_3 Motordrehzahl	[U/min]
X_4 Messwert Lamba-Sonde	[-]
X_5 Relativer Wandfilminhalt	[-]

#### OUTPUTS

Y_1 Motordrehzahl	[U/min]
Y_2 Messwert Lambda-Sonde	[-]

#### SCALING

U_1N, X_1N	1 Grad
U_2N, X_4N, X_5N, Y_2N	0.05
U_3N	1.6 Grad KW
X_2N	0.05 bar
X_3N, Y_1N	200 U/min

### 1.1.2 Boeing707

```
sys = Boeing707 () [Function File]
```

Creates a linearized state-space model of a Boeing 707-321 aircraft at  $v=80$  m/s ( $M = 0.26$ ,  $G_{a0} = -3^{circ}$ ,  $\pi lpha_0 = 4^{circ}$ ,  $kappa = 50^{circ}$ ).

System inputs: (1) thrust and (2) elevator angle.

System outputs: (1) airspeed and (2) pitch angle.

**Reference:** R. Brockhaus: *Flugregelung* (Flight Control), Springer, 1994.

### 1.1.3 MDSSystem

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Robust Control of a Mass-Damper-Spring System Lukas Reichlin August 2011

=====

Reference: Gu, D.W., Petkov, P.Hr. and Konstantinov, M.M. Robust Control Design with Matlab, Springer 2005

=====

### 1.1.4 optiPID

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optiPID Lukas Reichlin July 2009

=====

Numerical Optimization of an A/H PID Controller Required OCTAVE Packages: control, optim (and its dependencies) Required MATLAB Toolboxes: Control, Optimization

=====

### 1.1.5 WestlandLynx

`sys = WestlandLynx ()`

[Function File]

Model of the Westland Lynx Helicopter about hover.

#### INPUTS

main rotor collective  
longitudinal cyclic  
lateral cyclic  
tail rotor collective

#### STATES

pitch attitude	theta	[rad]
roll attitude	phi	[rad]
roll rate (body-axis)	p	[rad/s]
pitch rate (body-axis)	q	[rad/s]
yaw rate	xi	[rad/s]
forward velocity	v_x	[ft/s]
lateral velocity	v_y	[ft/s]
vertical velocity	v_z	[ft/s]

#### OUTPUTS

heave velocity	H_dot	[ft/s]
pitch attitude	theta	[rad]
roll attitude	phi	[rad]
heading rate	psi_dot	[rad/s]
roll rate	p	[rad/s]
pitch rate	q	[rad/s]

#### Reference:

Skogestad, S. and Postlethwaite I.

Multivariable Feedback Control: Analysis and Design

Second Edition

Wiley 2005

[http://www.nt.ntnu.no/users/skoge/book/2nd\\_edition/matlab\\_m/matfiles.htm](http://www.nt.ntnu.no/users/skoge/book/2nd_edition/matlab_m/matfiles.htm)

## 1.2 Linear Time Invariant Models

### 1.2.1 dss

`sys = dss (sys)` [Function File]  
`sys = dss (d)` [Function File]  
`sys = dss (a, b, c, d, e, ...)` [Function File]  
`sys = dss (a, b, c, d, e, tsam, ...)` [Function File]

Create or convert to descriptor state-space model.

#### Inputs

`sys` LTI model to be converted to state-space.  
`a` State transition matrix (n-by-n).  
`b` Input matrix (n-by-m).  
`c` Measurement matrix (p-by-n).  
`d` Feedthrough matrix (p-by-m).  
`e` Descriptor matrix (n-by-n).  
`tsam` Sampling time in seconds. If `tsam` is not specified, a continuous-time model is assumed.  
`...` Optional pairs of properties and values. Type `set (dss)` for more information.

#### Outputs

`sys` Descriptor state-space model.

#### Equations

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} \\ \mathbf{y} &= \mathbf{C} \mathbf{x} + \mathbf{D} \mathbf{u} \end{aligned}$$

`ss`, `tf`

### 1.2.2 frd

`sys = frd (sys)` [Function File]  
`sys = frd (sys, w)` [Function File]  
`sys = frd (H, w, ...)` [Function File]  
`sys = frd (H, w, tsam, ...)` [Function File]

Create or convert to frequency response data.

#### Inputs

`sys` LTI model to be converted to frequency response data. If second argument `w` is omitted, the interesting frequency range is calculated by the zeros and poles of `sys`.  
`H` Frequency response array (p-by-m-by-lw). In the SISO case, a vector (lw-by-1) or (1-by-lw) is accepted as well.  
`w` Frequency vector (lw-by-1) in radian per second [rad/s].  
`tsam` Sampling time in seconds. If `tsam` is not specified, a continuous-time model is assumed.  
`...` Optional pairs of properties and values. Type `set (frd)` for more information.

**Outputs**

*sys*            Frequency response data object.

dss, ss, tf

**1.2.3 ss**

*sys* = **ss** (*sys*) [Function File]

*sys* = **ss** (*d*) [Function File]

*sys* = **ss** (*a*, *b*) [Function File]

*sys* = **ss** (*a*, *b*, *c*) [Function File]

*sys* = **ss** (*a*, *b*, *c*, *d*, ...) [Function File]

*sys* = **ss** (*a*, *b*, *c*, *d*, *tsam*, ...) [Function File]

Create or convert to state-space model.

**Inputs**

*sys*            LTI model to be converted to state-space.

*a*            State transition matrix (n-by-n).

*b*            Input matrix (n-by-m).

*c*            Measurement matrix (p-by-n). If *c* is empty [] or not specified, an identity matrix is assumed.

*d*            Feedthrough matrix (p-by-m). If *d* is empty [] or not specified, a zero matrix is assumed.

*tsam*        Sampling time in seconds. If *tsam* is not specified, a continuous-time model is assumed.

...          Optional pairs of properties and values. Type **set** (**ss**) for more information.

**Outputs**

*sys*            State-space model.

**Example**



```

octave:1> a = [1 2 3; 4 5 6; 7 8 9];
octave:2> b = [10; 11; 12];
octave:3> stname = {"V", "A", "kJ"};
octave:4> sys = ss (a, b, [], [], "stname", stname)

```

```

sys.a =
      V      A      kJ
      V      1      2      3
      A      4      5      6
      kJ      7      8      9

```

```

sys.b =
      u1
      V    10
      A    11
      kJ    12

```

```

sys.c =
      V      A      kJ
      y1      1      0      0
      y2      0      1      0
      y3      0      0      1

```

```

sys.d =
      u1
      y1      0
      y2      0
      y3      0

```

Continuous-time model.

```
octave:5>
```

tf, dss

### 1.2.4 tf

<code>s = tf ("s")</code>	[Function File]
<code>z = tf ("z", tsam)</code>	[Function File]
<code>sys = tf (sys)</code>	[Function File]
<code>sys = tf (num, den, ...)</code>	[Function File]
<code>sys = tf (num, den, tsam, ...)</code>	[Function File]

Create or convert to transfer function model.

#### Inputs

<code>sys</code>	LTI model to be converted to transfer function.
<code>num</code>	Numerator or cell of numerators. Each numerator must be a row vector containing the exponents of the polynomial in descending order.
<code>den</code>	Denominator or cell of denominators. Each denominator must be a row vector containing the exponents of the polynomial in descending order.
<code>tsam</code>	Sampling time in seconds. If <code>tsam</code> is not specified, a continuous-time model is assumed.

... Optional pairs of properties and values. Type `set (tf)` for more information.

### Outputs

`sys` Transfer function model.

### Example

```
octave:1> s = tf ("s");
octave:2> G = 1/(s+1)
```

Transfer function "G" from input "u1" to output ...

$$y1: \frac{1}{s + 1}$$

```
octave:3> z = tf ("z", 0.2);
octave:4> H = 0.095/(z-0.9)
```

Transfer function "H" from input "u1" to output ...

$$y1: \frac{0.095}{z - 0.9}$$

```
Sampling time: 0.2 s
octave:5>
```

`ss`, `dss`

## 1.3 Model Data Access

### 1.3.1 @lti/dssdata

`[a, b, c, d, e, tsam] = dssdata (sys)` [Function File]  
`[a, b, c, d, e, tsam] = dssdata (sys, [])` [Function File]

Access descriptor state-space model data. Argument `sys` is not limited to descriptor state-space models. If `sys` is not a descriptor state-space model, it is converted automatically.

### Inputs

`sys` Any type of LTI model.

`[]` In case `sys` is not a dss model (descriptor matrix `e` empty), `dssdata (sys, [])` returns the empty element `e = []` whereas `dssdata (sys)` returns the identity matrix `e = eye (size (a))`.

### Outputs

`a` State transition matrix (n-by-n).

`b` Input matrix (n-by-m).

`c` Measurement matrix (p-by-n).

`d` Feedthrough matrix (p-by-m).

`e` Descriptor matrix (n-by-n).

`tsam` Sampling time in seconds. If `sys` is a continuous-time model, a zero is returned.

### 1.3.2 @lti/frdata

`[H, w, tsam] = frdata (sys)` [Function File]

`[H, w, tsam] = frdata (sys, "vector")` [Function File]

Access frequency response data. Argument `sys` is not limited to frequency response data objects. If `sys` is not a frd object, it is converted automatically.

#### Inputs

`sys` Any type of LTI model.

`"v", "vector"`

In case `sys` is a SISO model, this option returns the frequency response as a column vector (lw-by-1) instead of an array (p-by-m-by-lw).

#### Outputs

`H` Frequency response array (p-by-m-by-lw). In the SISO case, a vector (lw-by-1) is possible as well.

`w` Frequency vector (lw-by-1) in radian per second [rad/s].

`tsam` Sampling time in seconds. If `sys` is a continuous-time model, a zero is returned.

### 1.3.3 @lti/get

`get (sys)` [Function File]

`value = get (sys, "property")` [Function File]

Access property values of LTI objects.

### 1.3.4 @lti/set

`set (sys)` [Function File]

`sys = set (sys, "property", value)` [Function File]

Set or modify properties of LTI objects.

### 1.3.5 @lti/ssdata

`[a, b, c, d, tsam] = ssdata (sys)` [Function File]

Access state-space model data. Argument `sys` is not limited to state-space models. If `sys` is not a state-space model, it is converted automatically.

#### Inputs

`sys` Any type of LTI model.

#### Outputs

`a` State transition matrix (n-by-n).

`b` Input matrix (n-by-m).

`c` Measurement matrix (p-by-n).

`d` Feedthrough matrix (p-by-m).

`tsam` Sampling time in seconds. If `sys` is a continuous-time model, a zero is returned.

### 1.3.6 @lti/tfdata

`[num, den, tsam] = tfdata (sys)` [Function File]

`[num, den, tsam] = tfdata (sys, "tfpoly")` [Function File]

Access transfer function data.

## 1.4 Model Conversions

### 1.4.1 @lti/c2d

`sys = c2d (sys, tsam)` [Function File]  
`sys = c2d (sys, tsam, method)` [Function File]  
`sys = c2d (sys, tsam, "prewarp", w0)` [Function File]

Convert the continuous lti model into its discrete-time equivalent.

#### Inputs

`sys` Continuous-time LTI model.  
`tsam` Sampling time in seconds.  
`method` Optional conversion method. If not specified, default method "zoh" is taken.  
"zoh" Zero-order hold or matrix exponential.  
"tustin", "bilinear" Bilinear transformation or Tustin approximation.  
"prewarp" Bilinear transformation with pre-warping at frequency  $w0$ .

#### Outputs

`sys` Discrete-time LTI model.

### 1.4.2 @lti/d2c

`sys = d2c (sys)` [Function File]  
`sys = d2c (sys, method)` [Function File]  
`sys = d2c (sys, "prewarp", w0)` [Function File]

Convert the discrete lti model into its continuous-time equivalent.

#### Inputs

`sys` Discrete-time LTI model.  
`method` Optional conversion method. If not specified, default method "zoh" is taken.  
"zoh" Zero-order hold or matrix logarithm.  
"tustin", "bilinear" Bilinear transformation or Tustin approximation.  
"prewarp" Bilinear transformation with pre-warping at frequency  $w0$ .

#### Outputs

`sys` Continuous-time LTI model.

### 1.4.3 @lti/prescale

`[scaledsys, info] = prescale (sys)` [Function File]

Prescale state-space model. Uses SLICOT TB01ID and TG01AD by courtesy of NICONET e.V. (<http://www.slicot.org>). Frequency response commands perform automatic scaling unless model property *scaled* is set to *true*.

#### Inputs

`sys` LTI model.

#### Outputs

*scaledsys*    Scaled state-space model.

*info*        Structure containing additional information.

*info.SL*     Left scaling factors.  $Tl = \text{diag}(\text{info.SL})$ .

*info.SR*     Right scaling factors.  $Tr = \text{diag}(\text{info.SR})$ .

### Equations

$$\begin{aligned} Es &= Tl \ E \ Tr \\ As &= Tl \ A \ Tr \\ Bs &= Tl \ B \\ Cs &= \quad C \ Tr \\ Ds &= \quad D \end{aligned}$$

For proper state-space models,  $Tl$  and  $Tr$  are inverse of each other.

### 1.4.4 @lti/xperm

*sys* = `xperm` (*sys*, *st\_idx*) [Function File]  
 Reorder states in state-space models.

## 1.5 Model Interconnections

### 1.5.1 @lti/append

*sys* = `append` (*sys1*, *sys2*) [Function File]  
 Group LTI models by appending their inputs and outputs.

### 1.5.2 @lti/blkdiag

*sys* = `blkdiag` (*sys1*, *sys2*) [Function File]  
 Block-diagonal concatenation of LTI models.

### 1.5.3 @lti/connect

*sys* = `connect` (*sys*, *cm*, *inputs*, *outputs*) [Function File]  
 Arbitrary interconnections between the inputs and outputs of an LTI model.

### 1.5.4 @lti/feedback

*sys* = `feedback` (*sys1*) [Function File]  
*sys* = `feedback` (*sys1*, "+") [Function File]  
*sys* = `feedback` (*sys1*, *sys2*) [Function File]  
*sys* = `feedback` (*sys1*, *sys2*, "+") [Function File]  
*sys* = `feedback` (*sys1*, *sys2*, *feedin*, *feedout*) [Function File]  
*sys* = `feedback` (*sys1*, *sys2*, *feedin*, *feedout*, "+") [Function File]  
 Feedback connection of two LTI models.

### Inputs

*sys1*        LTI model of forward transmission.  $[p1, m1] = \text{size}(\text{sys1})$ .

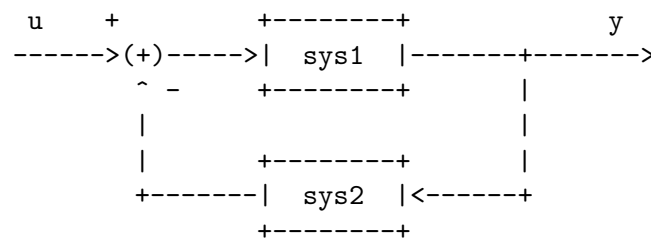
*sys2*        LTI model of backward transmission. If not specified, an identity matrix of appropriate size is taken.

<i>feedin</i>	Vector containing indices of inputs to <i>sys1</i> which are involved in the feedback loop. The number of <i>feedin</i> indices and outputs of <i>sys2</i> must be equal. If not specified, <i>1:m1</i> is taken.
<i>feedout</i>	Vector containing indices of outputs from <i>sys1</i> which are to be connected to <i>sys2</i> . The number of <i>feedout</i> indices and inputs of <i>sys2</i> must be equal. If not specified, <i>1:p1</i> is taken.
"+"	Positive feedback sign. If not specified, "-" for a negative feedback interconnection is assumed. +1 and -1 are possible as well, but only from the third argument onward due to ambiguity.

### Outputs

*sys*      Resulting LTI model.

### Block Diagram



### 1.5.5 @lft/lft

*sys* = lft (*sys1*, *sys2*) [Function File]  
*sys* = lft (*sys1*, *sys2*, *nu*, *ny*) [Function File]

Linear fractional transformation, also known as Redheffer star product.

### Inputs

*sys1*      Upper LTI model.

*sys2*      Lower LTI model.

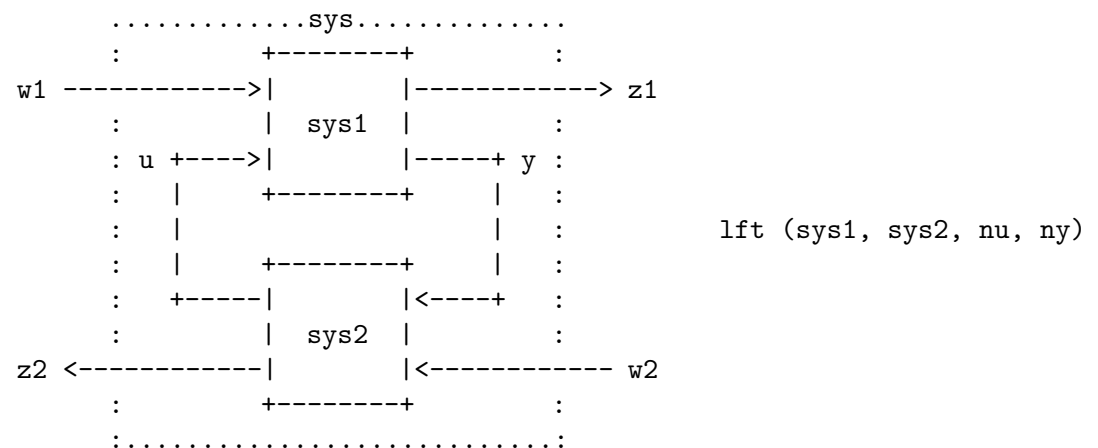
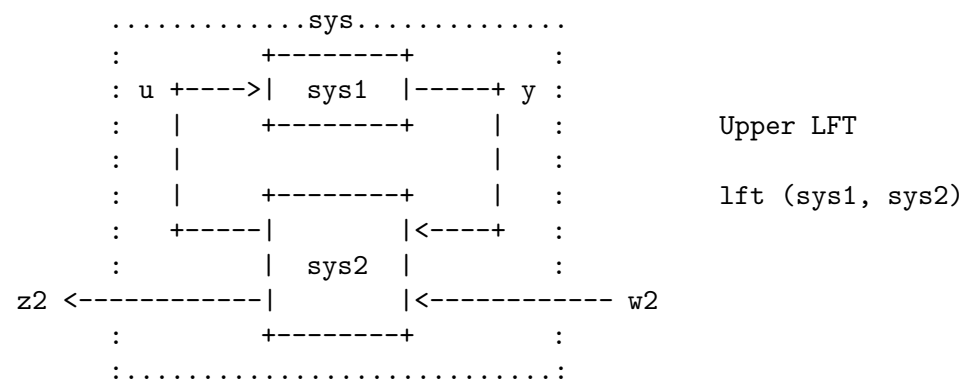
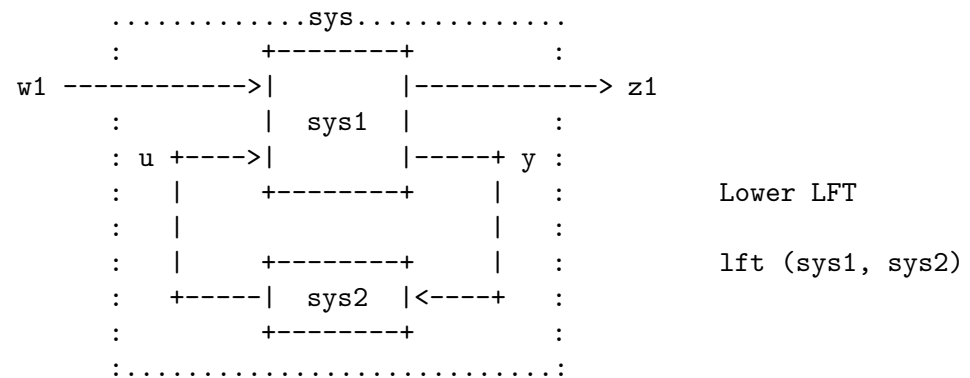
*nu*      The last *nu* inputs of *sys1* are connected with the first *nu* outputs of *sys2*. If not specified, min (*m1*, *p2*) is taken.

*ny*      The last *ny* outputs of *sys1* are connected with the first *ny* inputs of *sys2*. If not specified, min (*p1*, *m2*) is taken.

### Outputs

*sys*      Resulting LTI model.

### Block Diagram



### 1.5.6 @lti/mconnect

`sys = mconnect (sys, m)` [Function File]

`sys = mconnect (sys, m, inputs, outputs)` [Function File]

Arbitrary interconnections between the inputs and outputs of an LTI model.

#### Inputs

`sys` LTI system.

`m` Connection matrix. Each row belongs to an input and each column represents an output.

*inputs* Vector of indices of those inputs which are retained. If not specified, all inputs are kept.

*outputs* Vector of indices of those outputs which are retained. If not specified, all outputs are kept.

### Outputs

*sys* Interconnected system.

### Example

Solve the system equations of  
 $y(t) = G e(t)$   
 $e(t) = u(t) + M y(t)$   
 in order to build  
 $y(t) = H u(t)$   
 The matrix  $M$  for a (p-by-m) system  $G$   
 has  $m$  rows and  $p$  columns (m-by-p).

Example for a 3x2 system:

$u1 = -1*y1 + 5*y2 + 0*y3$

$u2 = \pi*y1 + 0*y2 - 7*y3$

$$M = \begin{bmatrix} -1 & 5 & 0 \\ \pi & 0 & 7 \end{bmatrix}$$

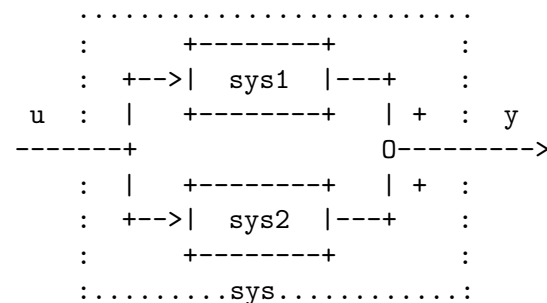
## 1.5.7 @lti/parallel

`sys = parallel (sys1, sys2)`

[Function File]

Parallel connection of two LTI systems.

### Block Diagram



`sys = parallel (sys1, sys2)`

## 1.5.8 @lti/series

`sys = series (sys1, sys2)`

[Function File]

`sys = series (sys1, sys2, outputs1, inputs2)`

[Function File]

Series connection of two LTI models.

### Block Diagram



```

.....
u  : +-----+ y1   u2 +-----+ : y
----->| sys1 |----->| sys2 |----->
      : +-----+           +-----+ :
      :.....sys.....

```

sys = series (sys1, sys2)

```

.....
      :               v2 +-----+ :
      :               ----->|      | : y
      : +-----+ y1   u2 | sys2 |----->
u  : |           |----->|      | :
----->| sys1 |      z1 +-----+ :
      : |           |----->      :
      : +-----+           :
      :.....sys.....

```

```

outputs1 = [1]
inputs2 = [2]
sys = series (sys1, sys2, outputs1, inputs2)

```

## 1.6 Model Characteristics

### 1.6.1 ctrb

`co = ctrb (sys)`

[Function File]

`co = ctrb (a, b)`

[Function File]

Return controllability matrix.

#### Inputs

`sys` LTI model.

`a` State transition matrix (n-by-n).

`b` Input matrix (n-by-m).

#### Outputs

`co` Controllability matrix.

#### Equation

$$C_o = [BABA^2B \dots A^{n-1}B]$$

### 1.6.2 @lti/dcgain

`k = dcgain (sys)`

[Function File]

DC gain of LTI model.

#### Inputs

`sys` LTI system.

#### Outputs

*k* DC gain matrix. For a system with *m* inputs and *p* outputs, the array *k* has dimensions [*p*, *m*].

freqresp

### 1.6.3 gram

*W* = gram (*sys*, *mode*) [Function File]

*Wc* = gram (*a*, *b*) [Function File]

gram (*sys*, "c") returns the controllability gramian of the (continuous- or discrete-time) system *sys*. gram (*sys*, "o") returns the observability gramian of the (continuous- or discrete-time) system *sys*. gram (*a*, *b*) returns the controllability gramian *Wc* of the continuous-time system  $dx/dt = ax + bu$ ; i.e., *Wc* satisfies  $aWc + mWc' + bb' = 0$ .

### 1.6.4 hsvd

*hsv* = hsvd (*sys*) [Function File]

*hsv* = hsvd (*sys*, "offset", *alpha*) [Function File]

Hankel singular values of the stable part of an LTI model. If no output arguments are given, the Hankel singular values are displayed in a plot. Uses SLICOT AB13AD by courtesy of NICONET e.V. (<http://www.slicot.org>)

### 1.6.5 @lti/isct

*bool* = isct (*sys*) [Function File]

Determine whether LTI model is a continuous-time system.

#### Inputs

*sys* LTI system.

#### Outputs

*bool* = 0 *sys* is a discrete-time system.

*bool* = 1 *sys* is a continuous-time system or a static gain.

### 1.6.6 isctrb

*bool* = isctrb (*sys*) [Function File]

*bool* = isctrb (*sys*, *tol*) [Function File]

*bool* = isctrb (*a*, *b*) [Function File]

*bool* = isctrb (*a*, *b*, *e*) [Function File]

*bool* = isctrb (*a*, *b*, [], *tol*) [Function File]

*bool* = isctrb (*a*, *b*, *e*, *tol*) [Function File]

Logical check for system controllability. Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Inputs

*sys* LTI model.

*a* State transition matrix.

*b* Input matrix.

*e* Descriptor matrix.

*tol* Optional roundoff parameter. Default value is 0.

#### Outputs

*bool* = 0    System is not controllable.

*bool* = 1    System is controllable.

isobsv

### 1.6.7 isdetectable

*bool* = isdetectable (*sys*) [Function File]  
*bool* = isdetectable (*sys*, *tol*) [Function File]  
*bool* = isdetectable (*a*, *c*) [Function File]  
*bool* = isdetectable (*a*, *c*, *e*) [Function File]  
*bool* = isdetectable (*a*, *c*, [], *tol*) [Function File]  
*bool* = isdetectable (*a*, *c*, *e*, *tol*) [Function File]  
*bool* = isdetectable (*a*, *c*, [], [], *dflg*) [Function File]  
*bool* = isdetectable (*a*, *c*, *e*, [], *dflg*) [Function File]  
*bool* = isdetectable (*a*, *c*, [], *tol*, *dflg*) [Function File]  
*bool* = isdetectable (*a*, *c*, *e*, *tol*, *dflg*) [Function File]

Logical test for system detectability. All unstable modes must be observable or all unobservable states must be stable. Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Inputs

*sys*            LTI system.

*a*              State transition matrix.

*c*              Measurement matrix.

*e*              Descriptor matrix.

*tol*            Optional tolerance for stability. Default value is 0.

*dflg* = 0    Matrices (*a*, *c*) are part of a continuous-time system. Default Value.

*dflg* = 1    Matrices (*a*, *c*) are part of a discrete-time system.

#### Outputs

*bool* = 0    System is not detectable.

*bool* = 1    System is detectable.

See `isstabilizable` for description of computational method. `isstabilizable`, `isstable`, `isctrb`, `isobsv`

### 1.6.8 @lti/isdt

*bool* = isdt (*sys*) [Function File]  
Determine whether LTI model is a discrete-time system.

#### Inputs

*sys*            LTI system.

#### Outputs

*bool* = 0    *sys* is a continuous-time system.

*bool* = 1    *sys* is a discrete-time system or a static gain.

### 1.6.9 @lti/isminimumphase

`bool = isminimumphase (sys)` [Function File]

`bool = isminimumphase (sys, tol)` [Function File]

Determine whether LTI system is minimum phase. If a square system  $P$  is minimum-phase, its inverse  $P^{-1}$  is stable.

### 1.6.10 isobsv

`bool = isobsv (sys)` [Function File]

`bool = isobsv (sys, tol)` [Function File]

`bool = isobsv (a, c)` [Function File]

`bool = isobsv (a, c, e)` [Function File]

`bool = isobsv (a, c, [], tol)` [Function File]

`bool = isobsv (a, c, e, tol)` [Function File]

Logical check for system observability. Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Inputs

`sys` LTI model.  
`a` State transition matrix.  
`c` Measurement matrix.  
`e` Descriptor matrix.  
`tol` Optional roundoff parameter. Default value is 0.

#### Outputs

`bool = 0` System is not observable.

`bool = 1` System is observable.

isctrb

### 1.6.11 @lti/issiso

`bool = issiso (sys)` [Function File]

Determine whether LTI model is single-input/single-output (SISO).

### 1.6.12 isstabilizable

`bool = isstabilizable (sys)` [Function File]

`bool = isstabilizable (sys, tol)` [Function File]

`bool = isstabilizable (a, b)` [Function File]

`bool = isstabilizable (a, b, e)` [Function File]

`bool = isstabilizable (a, b, [], tol)` [Function File]

`bool = isstabilizable (a, b, e, tol)` [Function File]

`bool = isstabilizable (a, b, [], [], dflg)` [Function File]

`bool = isstabilizable (a, b, e, [], dflg)` [Function File]

`bool = isstabilizable (a, b, [], tol, dflg)` [Function File]

`bool = isstabilizable (a, b, e, tol, dflg)` [Function File]

Logical check for system stabilizability. All unstable modes must be controllable or all uncontrollable states must be stable. Uses SLICOT AB01OD and TG01HD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Inputs



**1.6.16 @lti/pole**

`p = pole (sys)` [Function File]

Compute poles of LTI system.

**Inputs**

`sys` LTI model.

**Outputs**

`p` Poles of `sys`.

**1.6.17 pzmap**

`pzmap (sys)` [Function File]

`[p, z] = pzmap (sys)` [Function File]

Plot the poles and zeros of an LTI system in the complex plane. If no output arguments are given, the result is plotted on the screen. Otherwise, the poles and zeros are computed and returned.

**Inputs**

`sys` LTI model.

**Outputs**

`p` Poles of `sys`.

`z` Transmission zeros of `sys`.

**1.6.18 @lti/size**

`nvec = size (sys)` [Function File]

`n = size (sys, dim)` [Function File]

`[p, m] = size (sys)` [Function File]

LTI model size, i.e. number of outputs and inputs.

**Inputs**

`sys` LTI system.

`dim` If given a second argument, `size` will return the size of the corresponding dimension.

**Outputs**

`nvec` Row vector. The first element is the number of outputs (rows) and the second element the number of inputs (columns).

`n` Scalar value. The size of the dimension `dim`.

`p` Number of outputs.

`m` Number of inputs.

**1.6.19 @lti/zero**

`z = zero (sys)` [Function File]

`[z, k] = zero (sys)` [Function File]

Compute transmission zeros and gain of LTI model.

**Inputs**

`sys` LTI model.

## Outputs

$z$	Transmission zeros of sys.
-----	----------------------------

$k$	Gain of sys.
0	0.0000
1	0.0000
2	0.0000
3	0.0000
4	0.0000
5	0.0000
6	0.0000
7	0.0000
8	0.0000
9	0.0000
10	0.0000
11	0.0000
12	0.0000
13	0.0000
14	0.0000
15	0.0000
16	0.0000
17	0.0000
18	0.0000
19	0.0000
20	0.0000
21	0.0000
22	0.0000
23	0.0000
24	0.0000
25	0.0000
26	0.0000
27	0.0000
28	0.0000
29	0.0000
30	0.0000
31	0.0000
32	0.0000
33	0.0000
34	0.0000
35	0.0000
36	0.0000
37	0.0000
38	0.0000
39	0.0000
40	0.0000
41	0.0000
42	0.0000
43	0.0000
44	0.0000
45	0.0000
46	0.0000
47	0.0000
48	0.0000
49	0.0000
50	0.0000
51	0.0000
52	0.0000
53	0.0000
54	0.0000
55	0.0000
56	0.0000
57	0.0000
58	0.0000
59	0.0000
60	0.0000
61	0.0000
62	0.0000
63	0.0000
64	0.0000
65	0.0000
66	0.0000
67	0.0000
68	0.0000
69	0.0000
70	0.0000
71	0.0000
72	0.0000
73	0.0000
74	0.0000
75	0.0000
76	0.0000
77	0.0000
78	0.0000
79	0.0000
80	0.0000
81	0.0000
82	0.0000
83	0.0000
84	0.0000
85	0.0000
86	0.0000
87	0.0000
88	0.0000
89	0.0000
90	0.0000
91	0.0000
92	0.0000
93	0.0000
94	0.0000
95	0.0000
96	0.0000
97	0.0000
98	0.0000
99	0.0000

## 1.7 Model Simplification

### 1.7.1 @lti/minreal

`sys = minreal (sys)` [Function File]

```
sys = minreal (sys, tol) [Function File]
```

Minimal realization or zero-pole cancellation of LTI models.

### 1.7.2 @lti/sminreal

`sys = sminreal (sys)` [Function File]

```
sys = sminreal (sys, tol) [Function File]
```

Perform state-space model reduction based on structure. Remove states which have no influence on the input-output behaviour. The physical meaning of the states is retained.

## Inputs

*sys*      State-space model.

*tol* Optional tolerance for controllability and observability. Entries of the state-space matrices whose moduli are less or equal to *tol* are assumed to be zero. Default value is 0.

## Outputs

<i>sys</i>	Reduced state-space model.
------------	----------------------------

minreal

## 1.8 Time Domain Analysis

### 1.8.1 covar

`[p, q] = covar (sys, w)` [Function File]

Return the steady-state covariance.

## Inputs

*sys* LTI model.

Parameter	Value
$\mathbf{w}$	Intensity of white noise inputs which drive sys.

## Outputs

$p$  Output covariance.

$q$	State covariance.
-----	-------------------

lyap, dlyap

### 1.8.2 gensig

`[u, t] = gensig (sigtype, tau)` [Function File]  
`[u, t] = gensig (sigtype, tau, tfinal)` [Function File]  
`[u, t] = gensig (sigtype, tau, tfinal, tsam)` [Function File]

Generate periodic signal. Useful in combination with lsim.

#### Inputs

`sigtype = "sin"`

Sine wave.

`sigtype = "cos"`

Cosine wave.

`sigtype = "square"`

Square wave.

`sigtype = "pulse"`

Periodic pulse.

`tau` Duration of one period in seconds.

`tfinal` Optional duration of the signal in seconds. Default duration is 5 periods.

`tsam` Optional sampling time in seconds. Default spacing is  $\tau/64$ .

#### Outputs

`u` Vector of signal values.

`t` Time vector of the signal.

lsim

### 1.8.3 impulse

`[y, t, x] = impulse (sys)` [Function File]  
`[y, t, x] = impulse (sys, t)` [Function File]  
`[y, t, x] = impulse (sys, tfinal)` [Function File]  
`[y, t, x] = impulse (sys, tfinal, dt)` [Function File]

Impulse response of LTI system. If no output arguments are given, the response is printed on the screen.

#### Inputs

`sys` LTI model.

`t` Time vector. Should be evenly spaced. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

`tfinal` Optional simulation horizon. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

`dt` Optional sampling time. Be sure to choose it small enough to capture transient phenomena. If not specified, it is calculated by the poles of the system.

#### Outputs

`y` Output response array. Has as many rows as time samples (length of `t`) and as many columns as outputs.

`t` Time row vector.

`x` State trajectories array. Has **length (t)** rows and as many columns as states.

initial, lsim, step



### 1.8.4 initial

```
[y, t, x] = initial (sys, x0) [Function File]
[y, t, x] = initial (sys, x0, t) [Function File]
[y, t, x] = initial (sys, x0, tfinal) [Function File]
[y, t, x] = initial (sys, x0, tfinal, dt) [Function File]
```

Initial condition response of state-space model. If no output arguments are given, the response is printed on the screen.

#### Inputs

*sys* State-space model.

*x0* Vector of initial conditions for each state.

*t* Optional time vector. Should be evenly spaced. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

*tfinal* Optional simulation horizon. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.

*dt* Optional sampling time. Be sure to choose it small enough to capture transient phenomena. If not specified, it is calculated by the poles of the system.

#### Outputs

*y* Output response array. Has as many rows as time samples (length of *t*) and as many columns as outputs.

*t* Time row vector.

*x* State trajectories array. Has `length(t)` rows and as many columns as states.

#### Example

Continuous Time:  $\dot{x} = A x, \quad y = C x, \quad x(0) = x_0$

Discrete Time:  $x[k+1] = A x[k], \quad y[k] = C x[k], \quad x[0] = x_0$

impulse, lsim, step

### 1.8.5 lsim

```
[y, t, x] = lsim (sys, u) [Function File]
[y, t, x] = lsim (sys, u, t) [Function File]
[y, t, x] = lsim (sys, u, t, x0) [Function File]
[y, t, x] = lsim (sys, u, t, [], method) [Function File]
[y, t, x] = lsim (sys, u, t, x0, method) [Function File]
```

Simulate LTI model response to arbitrary inputs. If no output arguments are given, the system response is plotted on the screen.

#### Inputs

*sys* LTI model. System must be proper, i.e. it must not have more zeros than poles.

*u* Vector or array of input signal. Needs `length(t)` rows and as many columns as there are inputs. If *sys* is a single-input system, row vectors *u* of length `length(t)` are accepted as well.

<i>t</i>	Time vector. Should be evenly spaced. If <i>sys</i> is a continuous-time system and <i>t</i> is a real scalar, <i>sys</i> is discretized with sampling time <code>tsam = t/(rows(u)-1)</code> . If <i>sys</i> is a discrete-time system and <i>t</i> is not specified, vector <i>t</i> is assumed to be <code>0 : tsam : tsam*(rows(u)-1)</code> .
<i>x0</i>	Vector of initial conditions for each state. If not specified, a zero vector is assumed.
<i>method</i>	Discretization method for continuous-time models. Default value is <code>zoh</code> (zero-order hold). All methods from <code>c2d</code> are supported.

### Outputs

<i>y</i>	Output response array. Has as many rows as time samples (length of <i>t</i> ) and as many columns as outputs.
<i>t</i>	Time row vector. It is always evenly spaced.
<i>x</i>	State trajectories array. Has <code>length(t)</code> rows and as many columns as states.

impulse, initial, step

### 1.8.6 step

<code>[y, t, x] = step(sys)</code>	[Function File]
<code>[y, t, x] = step(sys, t)</code>	[Function File]
<code>[y, t, x] = step(sys, tfinal)</code>	[Function File]
<code>[y, t, x] = step(sys, tfinal, dt)</code>	[Function File]

Step response of LTI system. If no output arguments are given, the response is printed on the screen.

### Inputs

<i>sys</i>	LTI model.
<i>t</i>	Time vector. Should be evenly spaced. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.
<i>tfinal</i>	Optional simulation horizon. If not specified, it is calculated by the poles of the system to reflect adequately the response transients.
<i>dt</i>	Optional sampling time. Be sure to choose it small enough to capture transient phenomena. If not specified, it is calculated by the poles of the system.

### Outputs

<i>y</i>	Output response array. Has as many rows as time samples (length of <i>t</i> ) and as many columns as outputs.
<i>t</i>	Time row vector.
<i>x</i>	State trajectories array. Has <code>length(t)</code> rows and as many columns as states.

impulse, initial, lsim

## 1.9 Frequency Domain Analysis

### 1.9.1 bode

`[mag, pha, w] = bode (sys)` [Function File]

`[mag, pha, w] = bode (sys, w)` [Function File]

Bode diagram of frequency response. If no output arguments are given, the response is printed on the screen.

#### Inputs

*sys* LTI system. Must be a single-input and single-output (SISO) system.

*w* Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system.

#### Outputs

*mag* Vector of magnitude. Has length of frequency vector *w*.

*pha* Vector of phase. Has length of frequency vector *w*.

*w* Vector of frequency values used.

nichols, nyquist, sigma

### 1.9.2 bodemag

`[mag, w] = bodemag (sys)` [Function File]

`[mag, w] = bodemag (sys, w)` [Function File]

Bode magnitude diagram of frequency response. If no output arguments are given, the response is printed on the screen.

#### Inputs

*sys* LTI system. Must be a single-input and single-output (SISO) system.

*w* Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system.

#### Outputs

*mag* Vector of magnitude. Has length of frequency vector *w*.

*w* Vector of frequency values used.

bode, nichols, nyquist, sigma

### 1.9.3 @lti/freqresp

`H = freqresp (sys, w)` [Function File]

Evaluate frequency response at given frequencies.

#### Inputs

*sys* LTI system.

*w* Vector of frequency values.

#### Outputs

*H* Array of frequency response. For a system with *m* inputs and *p* outputs, the array *H* has dimensions [*p*, *m*, length (*w*)]. The frequency response at the frequency *w*(*k*) is given by *H*(:,:,*k*).

dcgain

### 1.9.4 margin

`[gamma, phi, w_gamma, w_phi] = margin (sys)` [Function File]

`[gamma, phi, w_gamma, w_phi] = margin (sys, tol)` [Function File]

Gain and phase margin of a system. If no output arguments are given, both gain and phase margin are plotted on a bode diagram. Otherwise, the margins and their corresponding frequencies are computed and returned.

#### Inputs

*sys* LTI model. Must be a single-input and single-output (SISO) system.

*tol* Imaginary parts below *tol* are assumed to be zero. If not specified, default value `sqrt (eps)` is taken.

#### Outputs

*gamma* Gain margin (as gain, not dBs).

*phi* Phase margin (in degrees).

*w\_gamma* Frequency for the gain margin (in rad/s).

*w\_phi* Frequency for the phase margin (in rad/s).

#### Equations

## CONTINUOUS SYSTEMS

## Gain Margin

$$L(j\omega) = \bar{L}(j\omega) \quad \text{BTW: } \bar{L}(j\omega) = L(-j\omega) = \text{conj}(L(j\omega))$$

$$\frac{\text{num}(j\omega)}{\text{den}(j\omega)} = \frac{\text{num}(-j\omega)}{\text{den}(-j\omega)}$$

$$\text{num}(j\omega) \text{den}(-j\omega) = \text{num}(-j\omega) \text{den}(j\omega)$$

$$\text{imag}(\text{num}(j\omega) \text{den}(-j\omega)) = 0$$

$$\text{imag}(\text{num}(-j\omega) \text{den}(j\omega)) = 0$$

## Phase Margin

$$|L(j\omega)| = \frac{|\text{num}(j\omega)|}{|\text{den}(j\omega)|} = 1$$

$$z \bar{z} = \text{Re } z + j \text{Im } z$$

$$\frac{\text{num}(j\omega)}{\text{den}(j\omega)} * \frac{\text{num}(-j\omega)}{\text{den}(-j\omega)} = 1$$

$$\text{num}(j\omega) \text{num}(-j\omega) - \text{den}(j\omega) \text{den}(-j\omega) = 0$$

$$\text{real}(\text{num}(j\omega) \text{num}(-j\omega) - \text{den}(j\omega) \text{den}(-j\omega)) = 0$$

## DISCRETE SYSTEMS

## Gain Margin

$$L(z) = L(1/z) \quad \text{BTW: } z = e^{j\omega T} \rightarrow w = \frac{\log z}{j T}$$

$$\frac{\text{num}(z)}{\text{den}(z)} = \frac{\text{num}(1/z)}{\text{den}(1/z)}$$

$$\text{num}(z) \text{den}(1/z) - \text{num}(1/z) \text{den}(z) = 0$$

## Phase Margin

$$|L(z)| = \frac{|\text{num}(z)|}{|\text{den}(z)|} = 1$$

$$L(z) L(1/z) = 1$$

$$\frac{\text{num}(z)}{\text{den}(z)} * \frac{\text{num}(1/z)}{\text{den}(1/z)} = 1$$

$$\text{num}(z) \text{num}(1/z) - \text{den}(z) \text{den}(1/z) = 0$$

PS: How to get  $L(1/z)$

roots

### 1.9.5 nichols

`[mag, pha, w] = nichols (sys)` [Function File]

`[mag, pha, w] = nichols (sys, w)` [Function File]

Nichols chart of frequency response. If no output arguments are given, the response is printed on the screen.

#### Inputs

*sys* LTI system. Must be a single-input and single-output (SISO) system.

*w* Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system.

#### Outputs

*mag* Vector of magnitude. Has length of frequency vector *w*.

*pha* Vector of phase. Has length of frequency vector *w*.

*w* Vector of frequency values used.

bode, nyquist, sigma

### 1.9.6 nyquist

`[re, im, w] = nyquist (sys)` [Function File]

`[re, im, w] = nyquist (sys, w)` [Function File]

Nyquist diagram of frequency response. If no output arguments are given, the response is printed on the screen.

#### Inputs

*sys* LTI system. Must be a single-input and single-output (SISO) system.

*w* Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system.

#### Outputs

*re* Vector of real parts. Has length of frequency vector *w*.

*im* Vector of imaginary parts. Has length of frequency vector *w*.

*w* Vector of frequency values used.

bode, nichols, sigma

### 1.9.7 sigma

`[sv, w] = sigma (sys)` [Function File]

`[sv, w] = sigma (sys, w)` [Function File]

`[sv, w] = sigma (sys, [], ptype)` [Function File]

`[sv, w] = sigma (sys, w, ptype)` [Function File]

Singular values of frequency response. If no output arguments are given, the singular value plot is printed on the screen;

#### Inputs

*sys* LTI system. Multiple inputs and/or outputs (MIMO systems) make practical sense.

- w* Optional vector of frequency values. If *w* is not specified, it is calculated by the zeros and poles of the system.
- ptype = 0* Singular values of the frequency response *H* of system *sys*. Default Value.
- ptype = 1* Singular values of the frequency response *inv(H)*; i.e. inversed system.
- ptype = 2* Singular values of the frequency response *I + H*; i.e. inversed sensitivity (or return difference) if *H = P \* C*.
- ptype = 3* Singular values of the frequency response *I + inv(H)*; i.e. inversed complementary sensitivity if *H = P \* C*.

### Outputs

- sv* Array of singular values. For a system with *m* inputs and *p* outputs, the array *sv* has  $\min(m, p)$  rows and as many columns as frequency points *length(w)*. The singular values at the frequency *w(k)* are given by *sv(:,k)*.
- w* Vector of frequency values used.
- bodemag, svd

## 1.10 Pole Placement

### 1.10.1 place

- f* = place (*sys*, *p*) [Function File]
- f* = place (*a*, *b*, *p*) [Function File]
- [*f*, *nfp*, *nap*, *nup*] = place (*sys*, *p*, *alpha*) [Function File]
- [*f*, *nfp*, *nap*, *nup*] = place (*a*, *b*, *p*, *alpha*) [Function File]

Pole assignment for a given matrix pair (*A*,*B*) such that  $p = \text{eig}(A-B*F)$ . If parameter *alpha* is specified, poles with real parts (continuous-time) or moduli (discrete-time) below *alpha* are left untouched. Uses SLICOT SB01BD by courtesy of NICONET e.V. (<http://www.slicot.org>)

### Inputs

- sys* LTI system.
- a* State transition matrix (n-by-n) of a continuous-time system.
- b* Input matrix (n-by-m) of a continuous-time system.
- p* Desired eigenvalues of the closed-loop system state-matrix *A-B\*F*. *length(p) <= rows(A)*.
- alpha* Specifies the maximum admissible value, either for real parts or for moduli, of the eigenvalues of *A* which will not be modified by the eigenvalue assignment algorithm. *alpha*  $\geq 0$  for discrete-time systems.

### Outputs

- f* State feedback gain matrix.
- nfp* The number of fixed poles, i.e. eigenvalues of *A* having real parts less than *alpha*, or moduli less than *alpha*. These eigenvalues are not modified by *place*.
- nap* The number of assigned eigenvalues. *nap = n-nfp-nup*.
- nup* The number of uncontrollable eigenvalues detected by the eigenvalue assignment algorithm.

**Note**

Place is also suitable to design estimator gains:

```
L = place (A.', C.', p).'  
L = place (sys.', p).'  
# useful for discrete-time systems
```

**1.10.2 rlocus**

`rlocus (sys)` [Function File]

`[rldata, k] = rlocus (sys[, increment, min_k, max_k])` [Function File]

Display root locus plot of the specified SISO system.

**Inputs**

*sys* LTI model. Must be a single-input and single-output (SISO) system.

*min\_k* Minimum value of *k*.

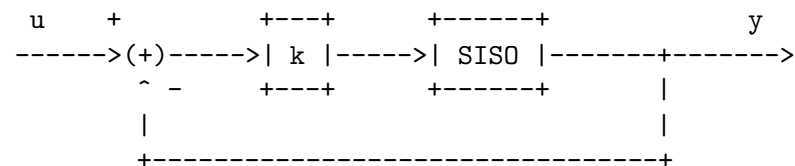
*max\_k* Maximum value of *k*.

*increment* The increment used in computing gain values.

**Outputs**

*rldata* Data points plotted: in column 1 real values, in column 2 the imaginary values.

*k* Gains for real axis break points.

**Block Diagram****1.11 Linear-Quadratic Control****1.11.1 dlqr**

`[g, x, l] = dlqr (sys, q, r)` [Function File]

`[g, x, l] = dlqr (sys, q, r, s)` [Function File]

`[g, x, l] = dlqr (a, b, q, r)` [Function File]

`[g, x, l] = dlqr (a, b, q, r, s)` [Function File]

`[g, x, l] = dlqr (a, b, q, r, [], e)` [Function File]

`[g, x, l] = dlqr (a, b, q, r, s, e)` [Function File]

Linear-quadratic regulator for discrete-time systems.

**Inputs**

*sys* Continuous or discrete-time LTI model.

*a* State transition matrix of discrete-time system.

*b* Input matrix of discrete-time system.

*q* State weighting matrix.

*r* Input weighting matrix.

*s* Optional cross term matrix. If *s* is not specified, a zero matrix is assumed.



*e* Optional descriptor matrix. If *e* is not specified, an identity matrix is assumed.

### Outputs

*g* State feedback matrix.

*x* Unique stabilizing solution of the discrete-time Riccati equation.

*l* Closed-loop poles.

### Equations

$$x[k+1] = A x[k] + B u[k], \quad x[0] = x_0$$

$$J(x_0) = \sum_{k=0}^{\infty} (x' Q x + u' R u + 2 x' S u)$$

$$L = \text{eig} (A - B*G)$$

dare, care, lqr

## 1.11.2 estim

`est = estim (sys, l)`

[Function File]

`est = estim (sys, l, sensors, known)`

[Function File]

Return state estimator for a given estimator gain.

### Inputs

*sys* LTI model.

*l* State feedback matrix.

*sensors* Indices of measured output signals *y* from *sys*. If omitted, all outputs are measured.

*known* Indices of known input signals *u* (deterministic) to *sys*. All other inputs to *sys* are assumed stochastic. If argument *known* is omitted, no inputs *u* are known.

### Outputs

*est* State-space model of estimator.

kalman, place

## 1.11.3 kalman

`[est, g, x] = kalman (sys, q, r)`

[Function File]

`[est, g, x] = kalman (sys, q, r, s)`

[Function File]

`[est, g, x] = kalman (sys, q, r, [], sensors, known)`

[Function File]

`[est, g, x] = kalman (sys, q, r, s, sensors, known)`

[Function File]

Design Kalman estimator for LTI systems.

### Inputs

*sys* Nominal plant model.

*q* Covariance of white process noise.

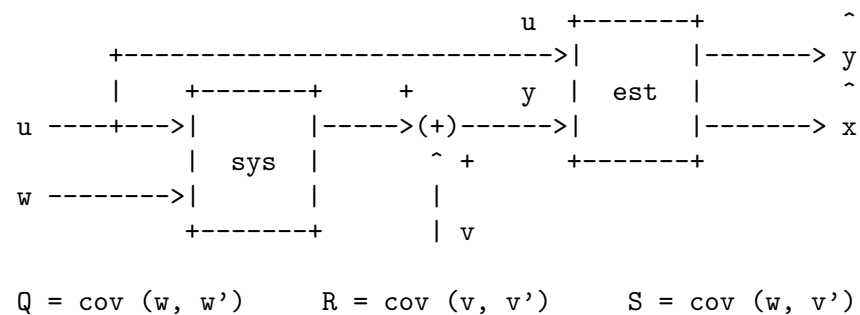
*r* Covariance of white measurement noise.

*s* Optional cross term covariance. Default value is 0.

<i>sensors</i>	Indices of measured output signals <i>y</i> from <i>sys</i> . If omitted, all outputs are measured.
<i>known</i>	Indices of known input signals <i>u</i> (deterministic) to <i>sys</i> . All other inputs to <i>sys</i> are assumed stochastic. If argument <i>known</i> is omitted, no inputs <i>u</i> are known.

**Outputs**

<i>est</i>	State-space model of the Kalman estimator.
<i>g</i>	Estimator gain.
<i>x</i>	Solution of the Riccati equation.

**Block Diagram**

care, dare, estim, lqr

**1.11.4 lqr**

$[g, x, l] = \text{lqr}(sys, q, r)$	[Function File]
$[g, x, l] = \text{lqr}(sys, q, r, s)$	[Function File]
$[g, x, l] = \text{lqr}(a, b, q, r)$	[Function File]
$[g, x, l] = \text{lqr}(a, b, q, r, s)$	[Function File]
$[g, x, l] = \text{lqr}(a, b, q, r, [], e)$	[Function File]
$[g, x, l] = \text{lqr}(a, b, q, r, s, e)$	[Function File]

Linear-quadratic regulator.

**Inputs**

<i>sys</i>	Continuous or discrete-time LTI model.
<i>a</i>	State transition matrix of continuous-time system.
<i>b</i>	Input matrix of continuous-time system.
<i>q</i>	State weighting matrix.
<i>r</i>	Input weighting matrix.
<i>s</i>	Optional cross term matrix. If <i>s</i> is not specified, a zero matrix is assumed.
<i>e</i>	Optional descriptor matrix. If <i>e</i> is not specified, an identity matrix is assumed.

**Outputs**

<i>g</i>	State feedback matrix.
<i>x</i>	Unique stabilizing solution of the continuous-time Riccati equation.
<i>l</i>	Closed-loop poles.

**Equations**

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}, \quad \mathbf{x}(0) = \mathbf{x}_0$$

$$J(\mathbf{x}_0) = \int_0^{\infty} (\mathbf{x}' \mathbf{Q} \mathbf{x} + \mathbf{u}' \mathbf{R} \mathbf{u} + \frac{1}{2} \mathbf{x}' \mathbf{S} \mathbf{u}) dt$$

$$\mathbf{L} = \text{eig}(\mathbf{A} - \mathbf{B} \mathbf{G})$$

care, dare, dlqr

## 1.12 Robust Control

### 1.12.1 augw

$P = \text{augw}(G, W1, W2, W3)$  [Function File]  
 Extend plant for stacked S/KS/T problem. Subsequently, the robust control problem can be solved by h2syn or hinfyn.

#### Inputs

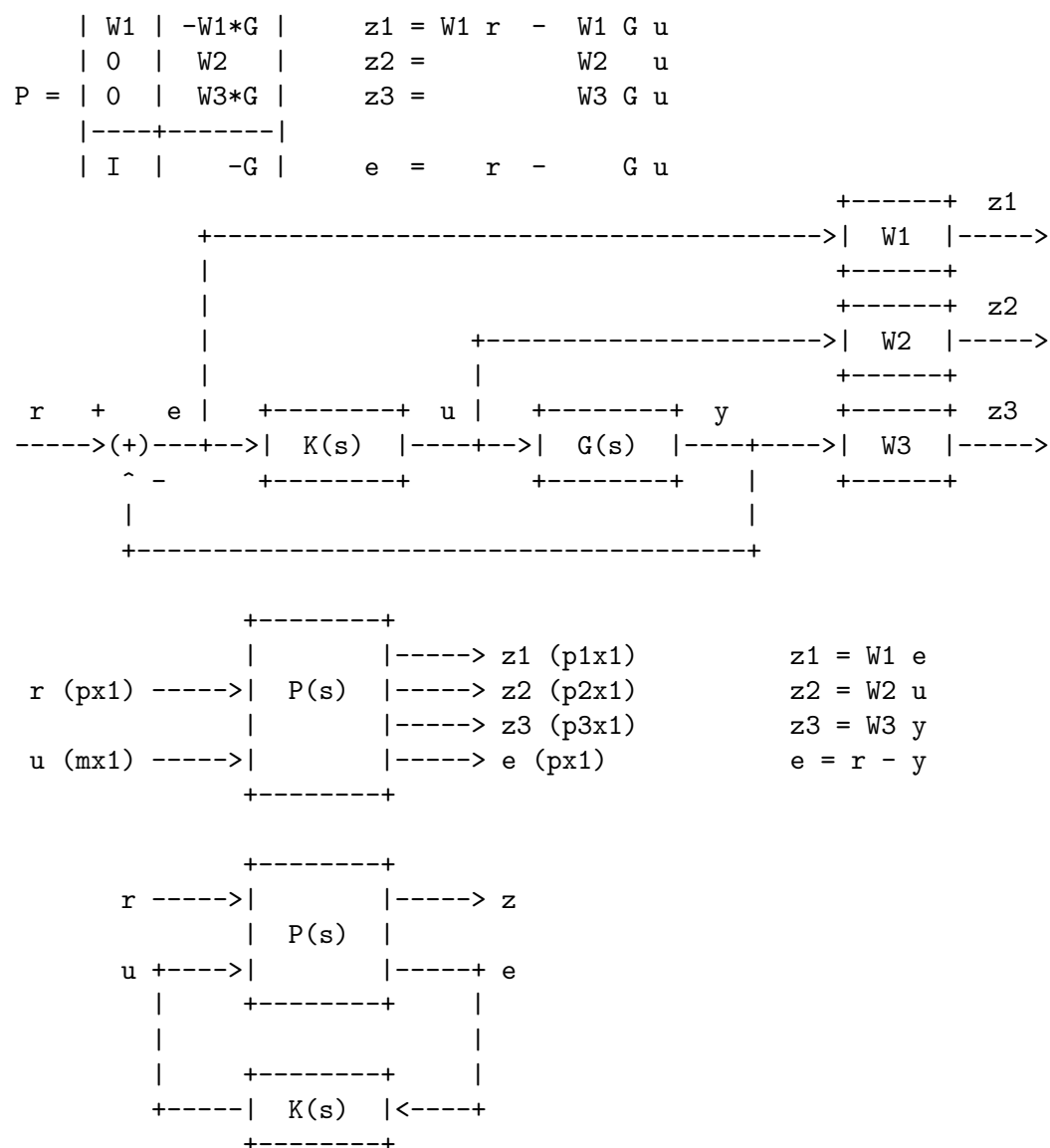
$G$	LTI model of plant.
$W1$	LTI model of performance weight. Bounds the largest singular values of sensitivity $S$ . Model must be empty [], SISO or of appropriate size.
$W2$	LTI model to penalize large control inputs. Bounds the largest singular values of $KS$ . Model must be empty [], SISO or of appropriate size.
$W3$	LTI model of robustness and noise sensitivity weight. Bounds the largest singular values of complementary sensitivity $T$ . Model must be empty [], SISO or of appropriate size.

All inputs must be proper/realizable. Scalars, vectors and matrices are possible instead of LTI models.

#### Outputs

$P$  State-space model of augmented plant.

#### Block Diagram



Reference:

Skogestad, S. and Postlethwaite I.

Multivariable Feedback Control: Analysis and Design  
Second Edition

Wiley 2005

Chapter 3.8: General Control Problem Formulation

## h2syn, hinfsyn, mixsyn

### 1.12.2 h2syn

`[K, N, gamma, rcond] = h2syn (P, nmeas, ncon)` [Function File]

H-2 control synthesis for LTI plant. Uses SLICOT SB10HD and SB10ED by courtesy of NICONET e.V. (<http://www.slicot.org>)

## Inputs

$P$  Generalized plant. Must be a proper/realizable LTI model.

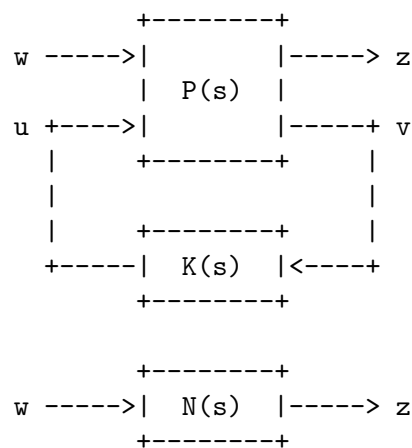
- nmeas*      Number of measured outputs  $v$ . The last  $nmeas$  outputs of  $P$  are connected to the inputs of controller  $K$ . The remaining outputs  $z$  (indices 1 to  $p-nmeas$ ) are used to calculate the H-2 norm.
- ncon*      Number of controlled inputs  $u$ . The last  $ncon$  inputs of  $P$  are connected to the outputs of controller  $K$ . The remaining inputs  $w$  (indices 1 to  $m-ncon$ ) are excited by a harmonic test signal.

### Outputs

- $K$       State-space model of the H-2 optimal controller.
- $N$       State-space model of the lower LFT of  $P$  and  $K$ .
- $\gamma$       H-2 norm of  $N$ .

### Block Diagram

$$\gamma = \min_K \|N(K)\|_2 \quad N = \text{lft}(P, K)$$



augw, lqr, dlqr, kalman

### 1.12.3 hinfsyn

$[K, N, \gamma, rcond] = \text{hinfsyn}(P, nmeas, ncon)$  [Function File]

$[K, N, \gamma, rcond] = \text{hinfsyn}(P, nmeas, ncon, gmax)$  [Function File]

H-infinity control synthesis for LTI plant. Uses SLICOT SB10FD and SB10DD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Inputs

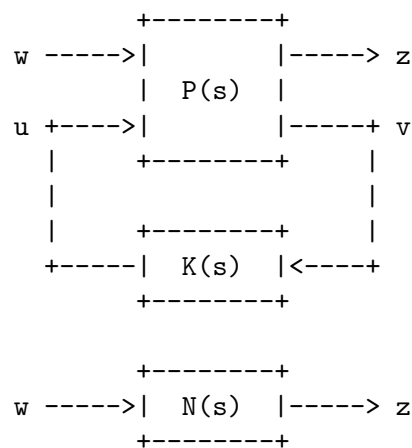
- $P$       Generalized plant. Must be a proper/realizable LTI model.
- nmeas*      Number of measured outputs  $v$ . The last  $nmeas$  outputs of  $P$  are connected to the inputs of controller  $K$ . The remaining outputs  $z$  (indices 1 to  $p-nmeas$ ) are used to calculate the H-infinity norm.
- ncon*      Number of controlled inputs  $u$ . The last  $ncon$  inputs of  $P$  are connected to the outputs of controller  $K$ . The remaining inputs  $w$  (indices 1 to  $m-ncon$ ) are excited by a harmonic test signal.
- $gmax$       The maximum value of the H-infinity norm of  $N$ . It is assumed that  $gmax$  is sufficiently large so that the controller is admissible.

**Outputs**

$K$  State-space model of the H-infinity (sub-)optimal controller.  
 $N$  State-space model of the lower LFT of  $P$  and  $K$ .  
 $\gamma$  L-infinity norm of  $N$ .

**Block Diagram**

$$\gamma = \min_K \|N(K)\|_{\infty} \quad N = \text{lft}(P, K)$$



augw, mixsyn

**1.12.4 mixsyn**

$[K, N, \gamma] = \text{mixsyn}(G, W1, W2, W3, \dots)$  [Function File]

Solve stacked S/KS/T H-inf problem. Bound the largest singular values of  $S$  (for performance),  $K S$  (to penalize large inputs) and  $T$  (for robustness and to avoid sensitivity to noise). In other words, the inputs  $r$  are excited by a harmonic test signal. Then the algorithm tries to find a controller  $K$  which minimizes the H-infinity norm calculated from the outputs  $z$ .

**Inputs**

$G$  LTI model of plant.  
 $W1$  LTI model of performance weight. Bounds the largest singular values of sensitivity  $S$ . Model must be empty  $[]$ , SISO or of appropriate size.  
 $W2$  LTI model to penalize large control inputs. Bounds the largest singular values of  $K S$ . Model must be empty  $[]$ , SISO or of appropriate size.  
 $W3$  LTI model of robustness and noise sensitivity weight. Bounds the largest singular values of complementary sensitivity  $T$ . Model must be empty  $[]$ , SISO or of appropriate size.  
 $\dots$  Optional arguments of `hinfsyn`. Type `help hinfsyn` for more information.

All inputs must be proper/realizable. Scalars, vectors and matrices are possible instead of LTI models.

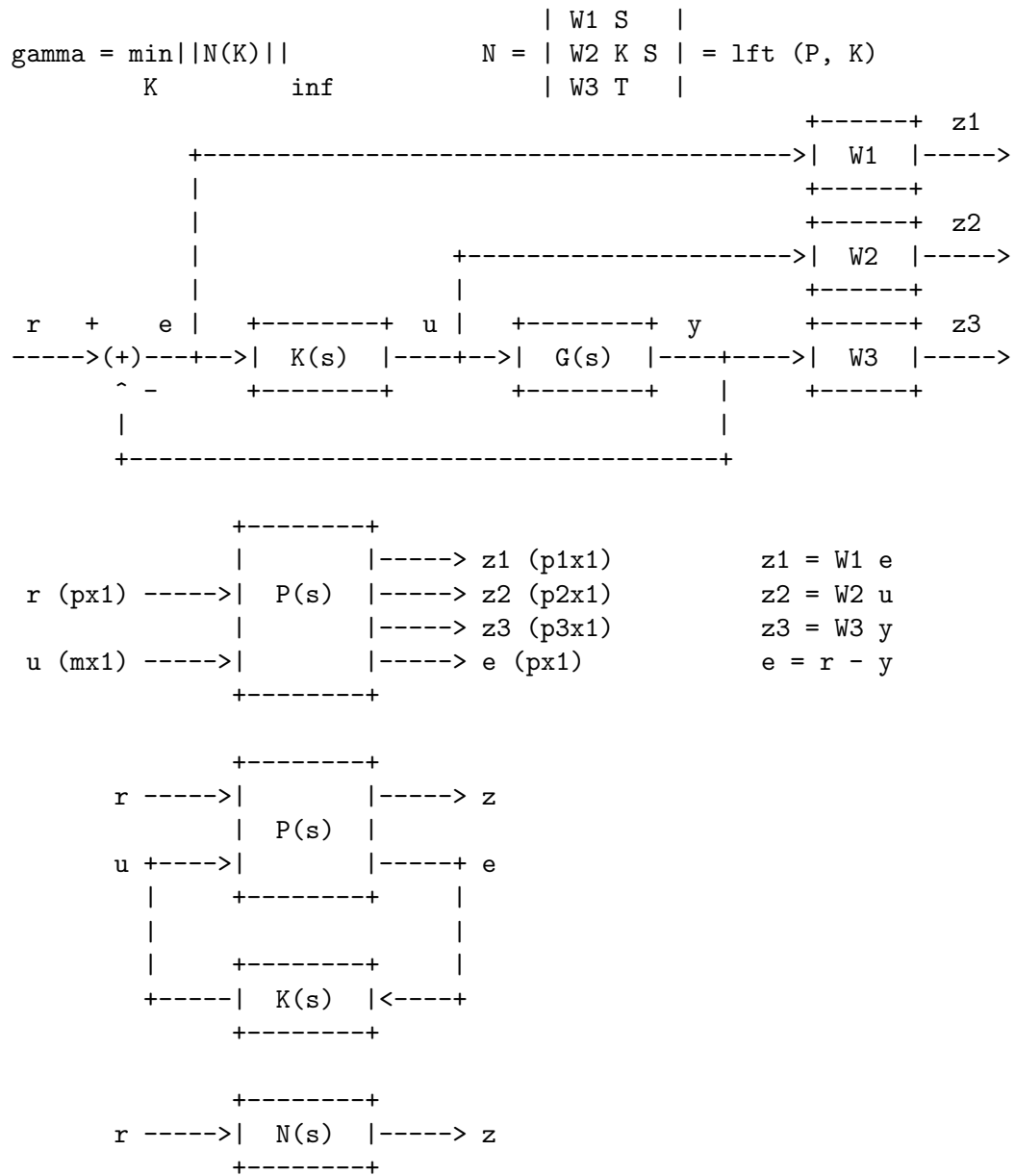
**Outputs**

$K$  State-space model of the H-infinity (sub-)optimal controller.

$N$             State-space model of the lower LFT of  $P$  and  $K$ .

$\gamma$             L-infinity norm of  $N$ .

**Block Diagram**



Extended Plant:  $P = \text{augw}(G, W1, W2, W3)$   
 Controller:  $K = \text{mixsyn}(G, W1, W2, W3)$   
 Entire System:  $N = \text{lft}(P, K)$   
 Open Loop:  $L = G * K$   
 Closed Loop:  $T = \text{feedback}(L)$

Reference:  
 Skogestad, S. and Postlethwaite I.  
 Multivariable Feedback Control: Analysis and Design  
 Second Edition  
 Wiley 2005  
 Chapter 3.8: General Control Problem Formulation



hinsyn, augw

### 1.12.5 ncfsyn

```
[K, N, gamma, info] = ncfsyn (G, W1, W2, factor) [Function File]
```

Normalized Coprime Factor (NCF) H-infinity synthesis. Compute positive feedback controller using the McFarlane/Glover Loop Shaping Design Procedure. Uses SLICOT SB10ID, SB10KD and SB10ZD by courtesy of NICONET e.V. (<http://www.slicot.org>)

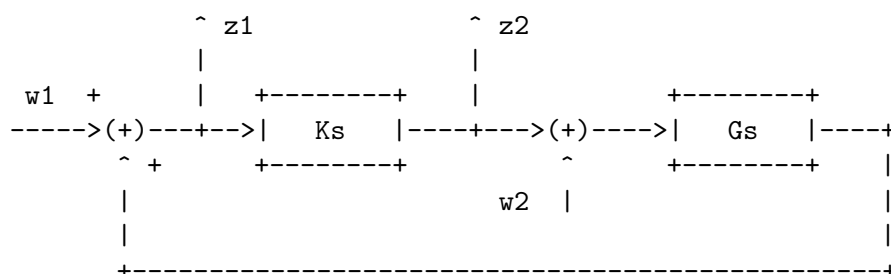
## Inputs

<i>G</i>	LTI model of plant.
<i>W1</i>	LTI model of precompensator. Model must be SISO or of appropriate size. An identity matrix is taken if <i>W1</i> is not specified or if an empty model [] is passed.
<i>W2</i>	LTI model of postcompensator. Model must be SISO or of appropriate size. An identity matrix is taken if <i>W2</i> is not specified or if an empty model [] is passed.
<i>factor</i>	<b>factor</b> = 1 implies that an optimal controller is required. <b>factor</b> > 1 implies that a suboptimal controller is required, achieving a performance that is <i>factor</i> times less than optimal. Default value is 1.

## Outputs

<i>K</i>	State-space model of the H-infinity loop-shaping controller.
<i>N</i>	State-space model of the closed loop depicted below.
<i>gamma</i>	L-infinity norm of <i>N</i> .
<i>info</i>	Structure containing additional information.
<i>info.emax</i>	Nugap robustness. <code>emax = inv (gamma)</code> .
<i>info.Gs</i>	Shaped plant. <code>Gs = W2 * G * W1</code> .
<i>info.Ks</i>	Controller for shaped plant. <code>Ks = ncfsyn (Gs)</code> .
<i>info.rcond</i>	Estimates of the reciprocal condition numbers of the Riccati equations.

### Block Diagram of N



## 1.13 Matrix Equation Solvers

### 1.13.1 care

`[x, l, g] = care (a, b, q, r)` [Function File]

[x, l, g] = care (a, b, q, r, s)	[Function File]
----------------------------------	-----------------

[x, l, g] = care (a, b, q, r, [], e) [Function File]

`[x, l, g] = care (a, b, q, r, s, e)` [Function File]  
 Solve continuous-time algebraic Riccati equation (ARE). Uses SLICOT SB02OD and SG02AD by courtesy of NICONET e.V. (<http://www.slicot.org>)

### Inputs

*a* Real matrix (n-by-n).  
*b* Real matrix (n-by-m).  
*q* Real matrix (n-by-n).  
*r* Real matrix (m-by-m).  
*s* Optional real matrix (n-by-m). If *s* is not specified, a zero matrix is assumed.  
*e* Optional descriptor matrix (n-by-n). If *e* is not specified, an identity matrix is assumed.

### Outputs

*x* Unique stabilizing solution of the continuous-time Riccati equation (n-by-n).  
*l* Closed-loop poles (n-by-1).  
*g* Corresponding gain matrix (m-by-n).

### Equations

$$A'X + XA - XB R^{-1} B'X + Q = 0$$

$$A'X + XA - (XB + S) R^{-1} (B'X + S') + Q = 0$$

$$G = R^{-1} B'X$$

$$G = R^{-1} (B'X + S')$$

$$L = \text{eig} (A - B*G)$$

$$A'XE + E'XA - E'XB R^{-1} B'XE + Q = 0$$

$$A'XE + E'XA - (E'XB + S) R^{-1} (B'XE + S') + Q = 0$$

$$G = R^{-1} B'XE$$

$$G = R^{-1} (B'XE + S)$$

$$L = \text{eig} (A - B*G, E)$$

dare, lqr, dlqr, kalman

### 1.13.2 dare

`[x, l, g] = dare (a, b, q, r)` [Function File]

`[x, l, g] = dare (a, b, q, r, s)` [Function File]

`[x, l, g] = dare (a, b, q, r, [], e)` [Function File]

`[x, l, g] = dare (a, b, q, r, s, e)` [Function File]

Solve discrete-time algebraic Riccati equation (ARE). Uses SLICOT SB02OD and SG02AD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Inputs

*a* Real matrix (n-by-n).

*b* Real matrix (n-by-m).

*q* Real matrix (n-by-n).

*r* Real matrix (m-by-m).

*s* Optional real matrix (n-by-m). If *s* is not specified, a zero matrix is assumed.

*e* Optional descriptor matrix (n-by-n). If *e* is not specified, an identity matrix is assumed.

#### Outputs

*x* Unique stabilizing solution of the discrete-time Riccati equation (n-by-n).

*l* Closed-loop poles (n-by-1).

*g* Corresponding gain matrix (m-by-n).

#### Equations

$$A'XA - X - A'XB (B'XB + R)^{-1} B'XA + Q = 0$$

$$A'XA - X - (A'XB + S) (B'XB + R)^{-1} (B'XA + S') + Q = 0$$

$$G = (B'XB + R)^{-1} B'XA$$

$$G = (B'XB + R)^{-1} (B'XA + S')$$

$$L = \text{eig} (A - B*G)$$

$$A'XA - E'XE - A'XB (B'XB + R)^{-1} B'XA + Q = 0$$

$$A'XA - E'XE - (A'XB + S) (B'XB + R)^{-1} (B'XA + S') + Q = 0$$

$$G = (B'XB + R)^{-1} B'XA$$

$$G = (B'XB + R)^{-1} (B'XA + S')$$

$$L = \text{eig} (A - B*G, E)$$

care, lqr, dlqr, kalman

### 1.13.3 dlyap

$x = \text{dlyap} (a, b)$  [Function File]  
 $x = \text{dlyap} (a, b, c)$  [Function File]  
 $x = \text{dlyap} (a, b, [], e)$  [Function File]

Solve discrete-time Lyapunov or Sylvester equations. Uses SLICOT SB03MD, SB04QD and SG03AD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Equations

$$AXA' - X + B = 0 \quad (\text{Lyapunov Equation})$$

$$AXB' - X + C = 0 \quad (\text{Sylvester Equation})$$

$$AXA' - EXE' + B = 0 \quad (\text{Generalized Lyapunov Equation})$$

dlyapchol, lyap, lyapchol

### 1.13.4 dlyapchol

$u = \text{dlyapchol} (a, b)$  [Function File]

`u = dlyapchol (a, b, e)` [Function File]  
 Compute Cholesky factor of discrete-time Lyapunov equations. Uses SLICOT SB03OD and SG03BD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Equations

$$A U' U A' - U' U + B B' = 0 \quad (\text{Lyapunov Equation})$$

$$A U' U A' - E U' U E' + B B' = 0 \quad (\text{Generalized Lyapunov Equation})$$

`dlyap`, `lyap`, `lyapchol`

### 1.13.5 lyap

`x = lyap (a, b)` [Function File]

`x = lyap (a, b, c)` [Function File]

`x = lyap (a, b, [], e)` [Function File]

Solve continuous-time Lyapunov or Sylvester equations. Uses SLICOT SB03MD, SB04MD and SG03AD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Equations

$$AX + XA' + B = 0 \quad (\text{Lyapunov Equation})$$

$$AX + XB + C = 0 \quad (\text{Sylvester Equation})$$

$$AXE' + EXA' + B = 0 \quad (\text{Generalized Lyapunov Equation})$$

`lyapchol`, `dlyap`, `dlyapchol`

### 1.13.6 lyapchol

`u = lyapchol (a, b)` [Function File]

`u = lyapchol (a, b, e)` [Function File]

Compute Cholesky factor of continuous-time Lyapunov equations. Uses SLICOT SB03OD and SG03BD by courtesy of NICONET e.V. (<http://www.slicot.org>)

#### Equations

$$A U' U + U' U A' + B B' = 0 \quad (\text{Lyapunov Equation})$$

$$A U' U E' + E U' U A' + B B' = 0 \quad (\text{Generalized Lyapunov Equation})$$

`lyap`, `dlyap`, `dlyapchol`

## 1.14 Overloaded Operators

### 1.14.1 @lti/horzcat

Horizontal concatenation of LTI objects. If necessary, object conversion is done by `sys_group`. Used by Octave for "[lti1, lti2]".

### 1.14.2 @lti/inv

Inversion of LTI objects.