

# Package ‘gaussratiovegind’

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**Title** Distribution of Gaussian Ratios

**Version** 2.0.3

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**Description** It is well known that the distribution of a Gaussian ratio does not follow a Gaussian distribution. The lack of awareness among users of vegetation indices about this non-Gaussian nature could lead to incorrect statistical modeling and interpretation. This package provides tools to accurately handle and analyse such ratios: density function, parameter estimation, simulation. An example on the study of chlorophyll fluorescence can be found in A. El Ghaziri et al. (2023) <[doi:10.3390/rs15020528](https://doi.org/10.3390/rs15020528)> and another method for parameter estimation is given in Bouhlel et al. (2023) <[doi:10.23919/EUSIPCO58844.2023.10290111](https://doi.org/10.23919/EUSIPCO58844.2023.10290111)>.

**License** GPL (>= 3)

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arabidopsis	<i>Statistics on Chlorophyll Fluorescence Parameters</i>
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### Description

Mean and standard deviation values on healthy and diseased tissues of chlorophyll fluorescence parameters  $F_0$  (minimum fluorescence) and  $F_m$  (maximum fluorescence) for a dataset of *Arabidopsis thaliana* plants infected with fungal pathogen data; parameters of the distribution of the ratio

$$\frac{F_v}{F_m} = \frac{F_m - F_0}{F_m}.$$

### Usage

arabidopsis

### Format

A data frame with 10 rows and 6 columns:

**time** times of the acquisition of chlorophyll fluorescence images

**condition** indicates if the plant was inoculated: healthy (inoculated with water) or diseased (inoculated with the pathogen)

**mF0, sF0** Mean and standard deviation values of the chlorophyll parameter  $F_0$

**mFm, sFm** Mean and standard deviation values of the chlorophyll parameter  $F_m$

**beta, rho, delta** the  $\beta$ ,  $\rho$  and  $\delta_y$  parameters of the distribution of  $\frac{F_v}{F_m} = \frac{F_m - F_0}{F_m}$  (distributed according to a normal ratio distribution, see Details)

### Details

On each leaf picture, the  $F_0$  and  $F_m$  values are normally distributed. Hence,  $\frac{F_0}{F_m}$  is a ratio of two normal distributions.

Let  $\mu_{F_0}$  and  $\sigma_{F_0}$  the mean and standard deviation of  $F_0$  and  $\mu_{F_m}$  and  $\sigma_{F_m}$  the mean and standard deviation of  $F_m$ . The parameters  $\beta$ ,  $\rho$  and  $\delta_y$  are given by:

$$\beta = \frac{\mu_{F_0}}{\mu_{F_m}}$$

$$\rho = \frac{\sigma_{F_m}}{\sigma_{F_0}}$$

$$\delta_y = \frac{\sigma_{F_m}}{\mu_{F_m}}$$

## References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:10.3390/rs15020528

Pavicic, M., Overmyer, K., Rehman, A.u., Jones, P., Jacobson, D., Himanen, K. Image-Based Methods to Score Fungal Pathogen Symptom Progression and Severity in Excised Arabidopsis Leaves. *Plants*, 10, 158 (2021). doi:10.3390/plants10010158

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 dnormratio

*Density Function of a Normal Ratio Distribution*


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## Description

Density of the ratio of two independent Gaussian distributions.

## Usage

dnormratio(z, bet, rho, delta)

## Arguments

z length  $p$  numeric vector.

bet, rho, delta numeric values. The parameters  $(\beta, \rho, \delta_y)$  of the distribution, see Details.

## Details

Let two independant random variables  $X \sim N(\mu_x, \sigma_x)$  and  $Y \sim N(\mu_y, \sigma_y)$ .

If we denote  $\beta = \frac{\mu_x}{\mu_y}$ ,  $\rho = \frac{\sigma_y}{\sigma_x}$  and  $\delta_y = \frac{\sigma_y}{\mu_y}$ , the probability distribution function of the ratio

$Z = \frac{X}{Y}$  is given by:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1 + \rho^2 z^2)} \left[ \exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2}\right) + \sqrt{\frac{\pi}{2}} q \operatorname{erf}\left(\frac{q}{\sqrt{2}}\right) \exp\left(-\frac{\rho^2(z - \beta)^2}{2\delta_y^2(1 + \rho^2 z^2)}\right) \right]$$

$$\text{with } q = \frac{1 + \beta\rho^2 z}{\delta_y \sqrt{1 + \rho^2 z^2}} \text{ and } \operatorname{erf}\left(\frac{q}{\sqrt{2}}\right) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{q}{\sqrt{2}}} \exp(-t^2) dt$$

Another expression of this density, used by the `estparnormratio()` function, is:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1 + \rho^2 z^2)} \exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2}\right) {}_1F_1\left(1, \frac{1}{2}; \frac{1}{2\delta_y^2} \frac{(1 + \beta\rho^2 z)^2}{1 + \rho^2 z^2}\right)$$

where  ${}_1F_1(a, b; x)$  is the confluent hypergeometric function (Kummer's function):

$${}_1F_1(a, b; x) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{x^n}{n!}$$

### Value

Numeric: the value of density.

### Author(s)

Pierre Santagostini, Angéline El Ghaziri, Nizar Bouhlel

### References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:10.3390/rs15020528

Marsaglia, G. 2006. Ratios of Normal Variables. *Journal of Statistical Software* 16. doi:10.18637/jss.v016.i04

Díaz-Francés, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. *Stat Papers* 54, 309–323 (2013). doi:10.1007/s0036201204292

### See Also

`pnormratio()`: probability distribution function.

`rnormratio()`: sample simulation.

`estparnormratio()`: parameter estimation.

### Examples

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
dnormratio(0, bet = beta1, rho = rho1, delta = delta1)
dnormratio(0.5, bet = beta1, rho = rho1, delta = delta1)
curve(dnormratio(x, bet = beta1, rho = rho1, delta = delta1), from = -0.1, to = 0.7)

# Second example
beta2 <- 2
rho2 <- 2
delta2 <- 2
dnormratio(0, bet = beta2, rho = rho2, delta = delta2)
```

```
dnormratio(0.5, bet = beta2, rho = rho2, delta = delta2)
curve(dnormratio(x, bet = beta2, rho = rho2, delta = delta2), from = -0.1, to = 0.7)
```

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estparnormratio      *Estimation of the Parameters of a Normal Ratio Distribution*

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### Description

Estimation of the parameters of a ratio  $Z = \frac{X}{Y}$ ,  $X$  and  $Y$  being two independent random variables distributed according to Gaussian distributions, using the EM (estimation-maximization) algorithm or variational inference. Depending on the estimation method, the `estparnormratio` function calls `estparEM` (EM algorithm) or `estparVB` (variational Bayes).

### Usage

```
estparnormratio(z, method = c("EM", "VB"), eps = 1e-06,
               display = FALSE, mux0 = 1, sigmax0 = 1,
               alphax0 = NULL, betax0 = NULL, muy0 = 1, sigmay0 = 1,
               alphay0 = NULL, betay0 = NULL)
```

```
estparEM(z, eps = 1e-06, display = FALSE, #plot = display,
         mux0 = 1, sigmax0 = 1, muy0 = 1, sigmay0 = 1)
```

```
estparVB(z, eps = 1e-06, display = FALSE, mux0 = 1, sigmax0 = 1,
         alphax0 = 1, betax0 = 1, muy0 = 1, sigmay0 = 1,
         alphay0 = 1, betay0 = 1)
```

```
estparEM(
  z,
  eps = 1e-06,
  display = FALSE,
  mux0 = 1,
  sigmax0 = 1,
  muy0 = 1,
  sigmay0 = 1
)
```

```
estparVB(
  z,
  eps = 1e-06,
  display = FALSE,
  mux0 = 1,
  sigmax0 = 1,
  alphax0 = 1,
  betax0 = 1,
  muy0 = 1,
  sigmay0 = 1,
  alphay0 = 1,
  betay0 = 1
)
```

```

    sigmay0 = 1,
    alphay0 = 1,
    betay0 = 1
)

```

### Arguments

**z** numeric.  
**method** the method used to estimate the parameters of the distribution. It can be "EM" (expectation-maximization) or "VB" (Variational Bayes).  
**eps** numeric. Precision for the estimation of the parameters (see Details).  
**display** logical. When TRUE the successive values of the stop criterion (distance between successive values) is printed.  
**mux0, sigmax0, muy0, sigmay0** initial values of the means and standard deviations of the  $X$  and  $Y$  variables. Default: mux0 = 1, sigmax0 = 1, muy0 = 1, sigmay0 = 1.  
**alphax0, betax0, alphay0, betay0** initial values for the variational Bayes method. Omitted if method="EM". If method="VB", if omitted, they are set to 1.

### Details

Let a random variable:  $Z = \frac{X}{Y}$ ,

$X$  and  $Y$  being normally distributed:  $X \sim N(\mu_x, \sigma_x)$  and  $Y \sim N(\mu_y, \sigma_y)$ .

The density probability of  $Z$  is:

$$f_Z(z; \beta, \rho, \delta_y) = \frac{\rho}{\pi(1 + \rho^2 z^2)} \exp\left(-\frac{\rho^2 \beta^2 + 1}{2\delta_y^2}\right) {}_1F_1\left(1, \frac{1}{2}; \frac{1}{2\delta_y^2} \frac{(1 + \beta\rho^2 z)^2}{1 + \rho^2 z^2}\right)$$

with:  $\beta = \frac{\mu_x}{\mu_y}$ ,  $\rho = \frac{\sigma_y}{\sigma_x}$ ,  $\delta_y = \frac{\sigma_y}{\mu_y}$ .

and  ${}_1F_1(a, b; x)$  is the confluent hypergeometric function (Kummer's function):

$${}_1F_1(a, b; x) = \sum_{n=0}^{+\infty} \frac{(a)_n}{(b)_n} \frac{x^n}{n!}$$

If method = "EM", the means and standard deviations  $\mu_x$ ,  $\sigma_x$ ,  $\mu_y$  and  $\sigma_y$  are estimated with the EM algorithm, as presented in El Ghaziri et al. If method = "VB", they are estimated with the variational Bayes method as presented in Bouhleb et al.

Then the parameters  $\beta$ ,  $\rho$ ,  $\delta_y$  of the  $Z$  distribution are computed from these means and standard deviations.

The estimation of  $\mu_x$ ,  $\sigma_x$ ,  $\mu_y$  and  $\sigma_y$  uses an iterative algorithm. The precision for their estimation is given by the eps parameter.

The computation uses the [kummer](#) function.

If there are ties in the z vector, it generates a warning, as there should be no ties in data distributed among a continuous distribution.

**Value**

A list of 3 elements beta, rho, delta: the estimated parameters of the  $Z$  distribution  $\hat{\beta}, \hat{\rho}, \hat{\delta}_y$ , with three attributes `attr(, "epsilon")` (precision of the result), `attr(, "k")` (number of iterations) and `attr(, "method")` (estimation method).

**Author(s)**

Pierre Santagostini, Angéline El Ghaziri, Nizar Bouhlel

**References**

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:10.3390/rs15020528

Bouhlel, N., Mercier, F., El Ghaziri, A., Rousseau, D., Parameter Estimation of the Normal Ratio Distribution with Variational Inference. 2023 31st European Signal Processing Conference (EU-SIPCO), Helsinki, Finland, 2023, pp. 1823-1827. doi:10.23919/EUSIPCO58844.2023.10290111

**See Also**

`dnormratio()`: probability density of a normal ratio.

`rnormratio()`: sample simulation.

**Examples**

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22

set.seed(1234)
z1 <- rnormratio(800, bet = beta1, rho = rho1, delta = delta1)

# With the EM method:
estparnormratio(z1, method = "EM")

# With the variational method:
estparnormratio(z1, method = "VB")

# Second example
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25

set.seed(1234)
z2 <- rnormratio(800, bet = beta2, rho = rho2, delta = delta2)

# With the EM method:
estparnormratio(z2, method = "EM")

# With the variational method:
```

```
estparnormratio(z2, method = "VB")
```

---

 kummer

*Confluent D-Hypergeometric Function*


---

### Description

Computes the Kummer's function, or confluent hypergeometric function.

### Usage

```
kummer(a, b, z, eps = 1e-06)
```

### Arguments

a	numeric.
b	numeric
z	numeric vector.
eps	numeric. Precision for the sum (default 1e-06).

### Details

The Kummer's confluent hypergeometric function is given by:

$${}_1F_1(a, b; z) = \sum_{n=0}^{+\infty} \frac{(a)_n z^n}{(b)_n n!}$$

where  $(z)_p$  is the Pochhammer symbol (see [pochhammer](#)).

The eps argument gives the required precision for its computation. It is the `attr(, "epsilon")` attribute of the returned value.

### Value

A numeric value: the value of the Kummer's function, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

### Author(s)

Pierre Santagostini, Angéline El Ghaziri, Nizar Bouhlel

### References

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). [doi:10.3390/rs15020528](https://doi.org/10.3390/rs15020528)



**Description**

Computes the logarithm of the Pochhammer symbol.

**Usage**

Inpochhammer(x, n)

**Arguments**

x	numeric.
n	positive integer.

**Details**

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)\dots(x+n-1)$$

So, if  $n > 0$ :

$$\log((x)_n) = \log(x) + \log(x+1) + \dots + \log(x+n-1)$$

If  $n = 0$ ,  $\log((x)_n) = \log(1) = 0$

**Value**

Numeric value. The logarithm of the Pochhammer symbol.

**Author(s)**

Pierre Santagostini, Nizar Bouhlel

**See Also**

[pochhammer](#), [kummer](#)

**Examples**

```
Inpochhammer(2, 0)
Inpochhammer(2, 1)
Inpochhammer(2, 3)
```

**Description**

Cumulative distribution of the ratio of two independent Gaussian distributions.

**Usage**

```
pnormratio(z, bet, rho, delta)
```

**Arguments**

`z` length  $p$  vector of quantiles.  
`bet, rho, delta` numeric values. The parameters  $(\beta, \rho, \delta_y)$  of the distribution, see Details.

**Details**

Let two independent random variables  $X \sim N(\mu_x, \sigma_x)$  and  $Y \sim N(\mu_y, \sigma_y)$ .

If we denote  $f_Z(z; \beta, \rho, \delta_y)$  the probability distribution function of the ratio  $Z = \frac{X}{Y}$ , with  $\beta = \frac{\mu_x}{\mu_y}$ ,  $\rho = \frac{\sigma_y}{\sigma_x}$  and  $\delta_y = \frac{\sigma_y}{\mu_y}$  (see `dnormratio()`, Details section).

The probability distribution for  $Z$  is given by:

$$F(z; \beta, \rho, \delta_y) = \int_{-\infty}^z f_Z(z; \beta, \rho, \delta_y)$$

This integral is computed using numerical integration.

**Value**

Numeric: the value of density.

**Author(s)**

Pierre Santagostini, Angéline El Ghaziri, Nizar Bouhlel

**References**

- El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:10.3390/rs15020528
- Marsaglia, G. 2006. Ratios of Normal Variables. *Journal of Statistical Software* 16. doi:10.18637/jss.v016.i04
- Díaz-Francés, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. *Stat Papers* 54, 309–323 (2013). doi:10.1007/s0036201204292

**See Also**

`dnormratio()`: density function.  
`rnormratio()`: sample simulation.  
`estparnormratio()`: parameter estimation.

**Examples**

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
pnormratio(0, bet = beta1, rho = rho1, delta = delta1)
pnormratio(0.5, bet = beta1, rho = rho1, delta = delta1)
curve(pnormratio(x, bet = beta1, rho = rho1, delta = delta1), from = -0.1, to = 0.7)

# Second example
beta2 <- 2
rho2 <- 2
delta2 <- 2
pnormratio(0, bet = beta2, rho = rho2, delta = delta2)
pnormratio(0.5, bet = beta2, rho = rho2, delta = delta2)
curve(pnormratio(x, bet = beta2, rho = rho2, delta = delta2), from = -0.1, to = 0.7)
```

---

pochhammer

*Pochhammer Symbol*


---

**Description**

Computes the Pochhammer symbol.

**Usage**

```
pochhammer(x, n)
```

**Arguments**

x                    numeric.  
n                    positive integer.

**Details**

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)\dots(x+n-1)$$

**Value**

Numeric value. The value of the Pochhammer symbol.

**Author(s)**

Pierre Santagostini, Nizar Bouhlel

**See Also**

[lnpochhammer](#), [kummer](#)

**Examples**

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

---

rnormratio

*Simulate from a Normal Ratio Distribution*

---

**Description**

Simulate data from a ratio of two independent Gaussian distributions.

**Usage**

```
rnormratio(n, bet, rho, delta)
```

**Arguments**

`n` integer. Number of observations. If `length(n) > 1`, the length is taken to be the number required.

`bet, rho, delta` numeric values. The parameters  $(\beta, \rho, \delta_y)$  of the distribution, see Details.

**Details**

Let two random variables  $X \sim N(\mu_x, \sigma_x)$  and  $Y \sim N(\mu_y, \sigma_y)$

with probability densities  $f_X$  and  $f_Y$ .

The parameters of the distribution of the ratio  $Z = \frac{X}{Y}$  are:  $\beta = \frac{\mu_x}{\mu_y}$ ,  $\rho = \frac{\sigma_y}{\sigma_x}$ ,  $\delta_y = \frac{\sigma_y}{\mu_y}$ .

$\mu_x$ ,  $\sigma_x$ ,  $\mu_y$  and  $\sigma_y$  are computed from  $\beta$ ,  $\rho$  and  $\delta_y$  (by fixing arbitrarily  $\mu_x = 1$ ) and two random samples  $(x_1, \dots, x_n)$  and  $(y_1, \dots, y_n)$  are simulated.

Then  $\left(\frac{x_1}{y_1}, \dots, \frac{x_n}{y_n}\right)$  is returned.

**Value**

A numeric vector: the produced sample.

**Author(s)**

Pierre Santagostini, Angéline El Ghaziri, Nizar Bouhlel

**References**

El Ghaziri, A., Bouhlel, N., Sapoukhina, N., Rousseau, D., On the importance of non-Gaussianity in chlorophyll fluorescence imaging. *Remote Sensing* 15(2), 528 (2023). doi:[10.3390/rs15020528](https://doi.org/10.3390/rs15020528)

Marsaglia, G. 2006. Ratios of Normal Variables. *Journal of Statistical Software* 16. doi:[10.18637/jss.v016.i04](https://doi.org/10.18637/jss.v016.i04)

Díaz-Francés, E., Rubio, F.J., On the existence of a normal approximation to the distribution of the ratio of two independent normal random variables. *Stat Papers* 54, 309–323 (2013). doi:[10.1007/s0036201204292](https://doi.org/10.1007/s0036201204292)

**See Also**

[dnormratio\(\)](#): probability density of a normal ratio.

[pnormratio\(\)](#): probability distribution function.

[estparnormratio\(\)](#): parameter estimation.

**Examples**

```
# First example
beta1 <- 0.15
rho1 <- 5.75
delta1 <- 0.22
rnormratio(20, bet = beta1, rho = rho1, delta = delta1)
```

```
# Second example
beta2 <- 0.24
rho2 <- 4.21
delta2 <- 0.25
rnormratio(20, bet = beta2, rho = rho2, delta = delta2)
```

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