

# Package ‘rankdifferencetest’

October 14, 2022

**Title** Kornbrot's Rank Difference Test

**Version** 2021-11-25

**Description** Implements Kornbrot's rank difference test as described in [doi:10.1111/j.2044-8317.1990.tb00939.x](https://doi.org/10.1111/j.2044-8317.1990.tb00939.x). This method is a modified Wilcoxon signed-rank test which produces consistent and meaningful results for ordinal or monotonically-transformed data.

**URL** <http://brettklamer.com/work/rankdifferencetest>

**License** MIT + file LICENSE

**LazyData** TRUE

**Depends** R (>= 3.6.0)

**Imports** Rdpack, coin, modeltools

**Suggests** testit, rmarkdown

**RdMacros** Rdpack

**RoxygenNote** 7.1.2

**Encoding** UTF-8

**NeedsCompilation** no

**Author** Brett Klamer [aut, cre] (<https://orcid.org/0000-0002-8334-4831>)

**Maintainer** Brett Klamer <[code@brettklamer.com](mailto:code@brettklamer.com)>

**Repository** CRAN

**Date/Publication** 2021-11-29 08:10:02 UTC

## R topics documented:

kornbrot_table1 . . . . .	2
rdt . . . . .	2
<b>Index</b>	<b>7</b>

---

kornbrot_table1	<i>Alertness example data</i>
-----------------	-------------------------------

---

### Description

An example dataset as seen in table 1 from Kornbrot (1990). The time/problem was recorded for each subject under placebo and drug conditions for the purpose of measuring 'alertness'.

### Usage

kornbrot\_table1

### Format

A data frame with 13 rows and 3 variables:

**subject** Subject identifier

**placebo** The time required to complete a task under the placebo condition

**drug** The time required to complete a task under the drug condition

### Details

The table 1 values appear to be rounded, thus results do not match exactly with further calculations in Kornbrot (1990).

### Source

Kornbrot DE (1990). "The rank difference test: A new and meaningful alternative to the Wilcoxon signed ranks test for ordinal data." *British Journal of Mathematical and Statistical Psychology*, 43(2), 241–264. ISSN 00071102, doi: [10.1111/j.20448317.1990.tb00939.x](https://doi.org/10.1111/j.20448317.1990.tb00939.x).

---

rdt	<i>Rank difference test</i>
-----	-----------------------------

---

### Description

Performs Kornbrot's rank difference test. It's a modified Wilcoxon signed-rank test which produces consistent and meaningful results for ordinal or monotonically transformed data.

**Usage**

```
rdt(
  data,
  formula,
  zero.method = c("Pratt", "Wilcoxon"),
  distribution = c("asymptotic", "approximate", "exact"),
  alternative = c("two.sided", "greater", "less"),
  return = c("data.frame", "coin"),
  ...
)
```

**Arguments**

data	A data.frame
formula	A formula of either form:  <b>y ~ x   block</b> For use when data is in long format. y is the numeric outcome, x is the binary grouping factor, and block is the observation-level grouping factor. x and block must be factors.  <b>y ~ x</b> For use when data is in wide format. y is the first measurement and x is the second measurement on the same observation.  The differences in ranks are calculated as y - x or, when x is a factor, the first factor level minus the second.
zero.method	A string for the method used to handle differences equal to zero: "Pratt" (default) or "Wilcoxon".
distribution	A string for the method used to calculate the conditional null distribution of the test statistic: Asymptotic distribution "asymptotic" (default), Monte Carlo resampling "approximate", or the exact distribution "exact".
alternative	A string for the alternative hypothesis: "two.sided" (default), "greater", or "less".
return	A string for the return object: "data.frame" (default) or "coin".
...	Further arguments passed to coin: <a href="#">:wilcoxsign_test</a> .

**Details**

For ordinal scale data, the Wilcoxon signed-rank test results in subtraction of those ordinal scale values. However, this subtraction is not meaningful on the ordinal scale. In addition, any monotone transformation of the data will result in different signed ranks, thus different p-values. However, ranking the original data allows for meaningful addition and subtraction of ranks and preserves ranks over monotonic transformation. Kornbrot described the rank difference test for this reason.

Kornbrot recommends that the rank difference procedure be used in preference to the Wilcoxon signed-rank test in all paired comparison designs where the data are not both of interval scale type and of known distribution type. The rank difference test preserves good power compared to Wilcoxon's signed-rank test, is more powerful than the sign test, and has the benefit of being a true distribution-free test.

The procedure for Wilcoxon's signed-rank test is as follows:

1. Calculate differences for each paired observation.
2. Remove differences equal to zero.
3. Order the absolute differences from smallest to largest.
4. Assign ranks  $1, \dots, n$  with average rank for ties.
5. Calculate  $W+ =$  sum of the ranks for positive differences. The sum of  $W+$  and  $W-$  is  $n(n + 1)/2$ , so either can be calculated from the other.
6. Choose the smaller of  $W+$  and  $W-$  as the test statistic  $W$ .
7. Since the test statistic is the smaller of  $W+$  and  $W-$ , the critical region is the left tail of the distribution.  $W$  is distributed approximately normal with mean  $\mu = (n(n + 1))/4$  and variance  $\sigma^2 = (Tn(n + 1)(2n + 1))/24$ , where  $T$  is a correction for ties and  $T = 1 - (\text{sum}(t^3 - t)/(N^3 - N))$ , summed over all ties, where  $t$  is the length of a tie. The continuity corrected mean  $\mu = ((n(n + 1))/4) + 0.5$ .

The procedure for Kornbrot's rank difference test is as follows:

1. Combine all  $2n$  observations.
2. Assign ranks  $1, \dots, 2n$  with average rank for ties.
3. Perform the Wilcoxon signed-rank test using the paired ranks.

The test statistic for the rank difference test ( $D$ ) is not exactly equal to the test statistic ( $W$ ) of the naive rank-transformed Wilcoxon signed-rank test (the latter being implemented in `rdt()`). Using  $W$  should result in a conservative estimate for  $D$ , and they approach in distribution as the sample size increases. Kornbrot (1990) discusses methods for calculating  $D$  when  $n < 7$  and  $8 < n \leq 20$ .

`zero.method = "Pratt"` uses the method by Pratt (1959), which first rank-transforms the absolute differences (including zeros) and then removes the ranks corresponding to zero-differences. `zero.method = "Wilcoxon"` uses the method by Wilcoxon (1950), which first removes the zero-differences and then rank-transforms the remaining absolute differences.

## Value

If `return = "data.frame"`, a `data.frame` with columns `p.value`, `z.statistic`, `formula`, `alternative`, and `method`.

If `return = "coin"`, an object inheriting from class `"IndependenceTest"`.

## References

Kornbrot DE (1990). "The rank difference test: A new and meaningful alternative to the Wilcoxon signed ranks test for ordinal data." *British Journal of Mathematical and Statistical Psychology*, **43**(2), 241–264. ISSN 00071102, doi: [10.1111/j.20448317.1990.tb00939.x](https://doi.org/10.1111/j.20448317.1990.tb00939.x).

Pratt JW (1959). "Remarks on Zeros and Ties in the Wilcoxon Signed Rank Procedures." *Journal of the American Statistical Association*, **54**(287), 655–667. ISSN 0162-1459, 1537-274X, doi: [10.1080/01621459.1959.10501526](https://doi.org/10.1080/01621459.1959.10501526).

Wilcoxon F (1950). "SOME RAPID APPROXIMATE STATISTICAL PROCEDURES." *Annals of the New York Academy of Sciences*, **52**(6), 808–814. ISSN 00778923, 17496632, doi: [10.1111/j.17496632.1950.tb53974.x](https://doi.org/10.1111/j.17496632.1950.tb53974.x).

**Examples**

```

#-----
# rdt() example
#-----
library(rankdifferencetest)

# Use example data from Kornbrot (1990)
data <- kornbrot_table1

# Create long-format data for demonstration purposes
data_long <- reshape(
  data = kornbrot_table1,
  direction = "long",
  varying = c("placebo", "drug"),
  v.names = c("time"),
  idvar = "subject",
  times = c("placebo", "drug"),
  timevar = "treatment",
  new.row.names = seq_len(prod(length(c("placebo", "drug")), nrow(kornbrot_table1)))
)
# Subject and treatment must be factors. The ordering of the treatment factor
# will determine the difference (placebo - drug).
data_long$subject <- factor(data_long$subject)
data_long$treatment <- factor(data_long$treatment, levels = c("placebo", "drug"))

# Recreate analysis and results from section 7.1 in Kornbrot (1990)
## The p-value shown in Kornbrot (1990) was continuity corrected. rdt() does
## not apply a continuity correction, so the p-value here will be slightly
## lower. It does match the uncorrected p-value shown in footnote on page 246.
rdt(
  data = data,
  formula = placebo ~ drug,
  alternative = "two.sided",
  distribution = "asymptotic"
)$p.value/2
rdt(
  data = data_long,
  formula = time ~ treatment | subject,
  alternative = "two.sided",
  distribution = "asymptotic"
)$p.value/2

# The same outcome is seen after transforming time to rate.
## The rate transformation inverts the rank ordering.
data$placebo_rate <- 60 / data$placebo
data$drug_rate <- 60 / data$drug
data_long$rate <- 60 / data_long$time

rdt(
  data = data,
  formula = placebo_rate ~ drug_rate,
  alternative = "two.sided",

```

```
    distribution = "asymptotic"
  )$p.value/2
rdt(
  data = data_long,
  formula = rate ~ treatment | subject,
  alternative = "two.sided",
  distribution = "asymptotic"
)$p.value/2

# In contrast to the rank difference test, the Wilcoxon signed-rank test
# produces differing results. See table 1 and table 2 in Kornbrot (1990).
wilcox.test(
  x = data$placebo,
  y = data$drug,
  paired = TRUE,
  exact = TRUE,
  alternative = "two.sided"
)$p.value/2
wilcox.test(
  x = data$placebo_rate,
  y = data$drug_rate,
  paired = TRUE,
  exact = TRUE,
  alternative = "two.sided"
)$p.value/2
```

# Index

\* **datasets**

kornbrot\_table1, [2](#)

IndependenceTest, [4](#)

kornbrot\_table1, [2](#)

rdt, [2](#)

wilcoxsigntest, [3](#)